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Measures of Distortion in Possibilistic Approximations of Consistent Random Sets and Intervals

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Abstract

The available of probability or possibility measures for random set (Dempster-Shafer evidence theoretical) structures are highly desirable. Probabilistic conditions involve disjointness or specificity, while possibilistic conditions involve consonance of the underlying focal elements. Consistency results in possibilistic distributions, but not measures, but then a unique approximation is available. Especially in random interval measurement situations, this condition is common. In this paper we develop some of the mathematical ideas necessary to develop a measure of the distortion introduced by this consonant approximation of a consistent random set.

1. Introduction

Random set measurement (mathematically equivalent to the derivation of Dempster-Shafer bodies of evidence from empirical sources) is a well-justified method for the empirical determinination of, on the one hand, specialized point-trace forms like possibility distributions and other normalized fuzzy sets like fuzzy numbers and intervals; and, on the other hand, specialized evidence measures (belief and plausibility measures) like possibility measures [5, 7].

An issue in the use of this method is the underlying class structure of the focal set of observed subsets. This structure can be completely general, or have a special structure such as specific (singleton subsets), disjoint, rings (intersecting subsets), consistent (global intersecting subsets), or consonant (completely nested subsets) [4]. Depending on the structure present, the resulting point traces and evidence measures have different properties. When consonant, the plausibility measure is naturally a possibility measure. In the more general case of consistency, while the plausibility measure is not a natural possibility measure, the trace is a natural possibility distribution, and a unique and well-justified possibilistic approximation of the plausibility measure is available. Finally, consistent approximations of inconsistent random sets are also available [6].

Consonant random sets provide a good empirical basis for applications of possibility theory. And yet, consistent but non-consonant random sets are a much more common case, for example in problems requiring the combination of interval measurements [3, 8]. Possibilistic approximations of these are not only well justified, they also bring great computational advantages, since they are representable in linear, rather than exponential, computational space and time. But of course such advantages are bought at the price of approximation: the "less consonant" the underlying consistent random set, the more the resulting approximated possibility measure distorts the empirical plausibility measure actually generated by the measured evidence.

In this paper we develop some of the mathematical ideas necessary to develop a measure of the distortion introduced by a consonant approximation of a consistent random set.

2. Random Sets and Fuzzy Measures

Given a finite universe $\Omega := \{\omega_i\}, 1 \leq i \leq n$ and evidence function (basic probability assignment) $m: 2^\Omega \mapsto [0,1]$, let $\mathcal{S} := \{\langle A_j, m_j \rangle : m_j > 0\}$ be a finite random set generated by m, where $\langle \cdot \rangle$ is a vector, $A_j \subseteq \Omega, m_j := m(A_j)$, and $1 \leq j \leq N := |\mathcal{S}| \leq 2^n - 1$. Let $\vec{m} := \langle m_j \rangle$ be an N-vector. Denote the focal set as $\mathcal{F}(\mathcal{S}) := \{A_j : m_j > 0\}$, where each A_j is a focal element, and the core as

$$\mathbf{C}(\mathcal{S}) = \mathbf{C}(\mathcal{F}(\mathcal{S})) := \bigcap_{A_j \in \mathcal{F}(\mathcal{S})} A_j.$$

$\mathcal{S}$ has belief and plausibility measures $\text{Bel}, \text{Pl}: 2^\Omega \mapsto [0,1]$, where $\forall A \subseteq \Omega$

$$\text{Bel}(A) = \sum_{A_j \subseteq A} m_j, \qquad \text{Pl}(A) = \sum_{A_j \not\perp A} m_j \quad (1)$$

and $A \perp B := A \cap B = \emptyset$. Given two random sets $\mathcal{S}, \mathcal{S}'$ on Ω, then random set inclusion is defined as [1] $\mathcal{S} \subseteq \mathcal{S}' := \forall A \subseteq \Omega, \text{Pl}(A) \leq \text{Pl}'(A)$.

Pl and Bel have the following linear algebraic formulations:

$$\vec{\text{Bel}} = \mathbf{G}^{\subseteq} \vec{m}^T, \quad \vec{\text{Pl}} = \mathbf{G}^{\cap} \vec{m}^T,$$

where:

- $\vec{\text{Bel}} := \langle \text{Bel}(A) \rangle$ and $\vec{\text{Pl}} := \langle \text{Pl}(A) \rangle$ are $2^n - 1$-vectors of the belief and plausibility measure values $\forall A \subseteq \Omega$;
- $\mathbf{G}^{\subseteq}$ and $\mathbf{G}^{\cap}$ are $2^n - 1 \times 2^n - 1$ matrices where for $1 \leq j, k \leq 2^n - 1$

$$\mathbf{G}^{\subseteq}_{j,k} = \begin{cases} 1, & A_j \subseteq A_k \\ 0, & \text{otherwise} \end{cases}$$

$$\mathbf{G}^{\cap}_{j,k} = \begin{cases} 1, & A_j \not\perp A_k \\ 0, & \text{otherwise} \end{cases}.$$

  Note that $\mathbf{G}^{\subseteq}$ can be derived as the tensor product

$$\mathbf{G}^{\subseteq} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^{\otimes n},$$

  and $\mathbf{G}^{\cap} = 1 - \mathbf{G}'^{\subseteq}$, where $\mathbf{G}'^{\subseteq}$ is $\mathbf{G}^{\subseteq}$ reflected through the columns.

Bel and Pl are fuzzy measures $\nu: 2^\Omega \mapsto [0,1]$, and are thereby monotonic with $A \subseteq B \rightarrow \nu(A) \leq \nu(B)$, and normal with $\nu(\Omega) = 1$. A fuzzy measure is decomposable in a conorm $\sqcup: [0,1]^2 \mapsto [0,1]$ (a commutative, monotonic, associative operator with identity 0) if

$$\nu(A \cup B) \sqcup \nu(A \cap B) = \nu(A) \sqcup \nu(B).$$

For any fuzzy measure ν, define the trace $\rho_\nu: \Omega \mapsto [0,1]$ as the singleton measure value $\rho_\nu(\omega_i) := \nu(\{\omega_i\})$. For a random set $\mathcal{S}$, denote its trace as $\rho^{\mathcal{S}} := \rho_{\text{Pl}}$, so that

$$\rho^{\mathcal{S}}(\omega_i) = \sum_{A_j \ni \omega_i} m_j. \quad (2)$$

Denote the n-vector $\vec{\rho} = \langle \rho^{\mathcal{S}}_i \rangle := \langle \rho^{\mathcal{S}}(\omega_i) \rangle$. Decomposability is equivalent to distributionality:

Theorem 1 [4] A fuzzy measure ν is decomposable in $\sqcup$ if and only if $\forall A \subseteq \Omega, \nu(A) = \bigsqcup_{\omega_i \in A} \rho_\nu(\omega_i)$

where we use operator notation for $\sqcup$ in virtue of its associativity. Note that normalization of ν results in normalization of the trace $\bigsqcup_{\omega_i \in \Omega} \rho(\omega_i) = 1$.

Any fuzzy measure determines its distribution: $\nu \rightarrow \rho_\nu$. Decomposable fuzzy measures are moreover uniquely determined by their distributions, so that $\nu \leftrightarrow \rho_\nu$. This is a great advantage computationally, since $|\nu|$ grows with 2^n, but $|\rho_\nu|$ grows only with n. Thus a distributional fuzzy measure can be characterized by vastly less information than its non-distributional cousins.

For a random set $\mathcal{S}$, define the numerical aggregator as the $n \times N$ matrix $\mathbf{H} := [h_{ij}]$, where $h_{ij} := \chi_{A_j}(\omega_i)$ and χ_{A_j} is the characteristic function of A_j, so that

$$\chi_{A_j}(\omega_i) := \begin{cases} 1, & \omega_i \in A_j \\ 0, & \omega_i \notin A_j \end{cases}, \quad A_j \in \mathcal{F}(\mathcal{S}), \quad \omega_i \in \Omega.$$

The j'th column of $\mathbf{H}$ is the vector representation of the characteristic function of A_j, so that $\mathbf{H}$ is a boolean matrix with no column repeated. Letting $\mathbf{H}(i)$ be the i'th row of $\mathbf{H}$, an N-vector, then

$$\rho_i = \mathbf{H}(i) \cdot \vec{m}^T, \qquad \vec{\rho}^T = \mathbf{H}\vec{m}^T \quad (3)$$

so that the left and right sides of (3) are the vector and matrix representations of (2) and (1) respectively.

We now introduce some example structures to be used below. Let $\Omega = \{x, y, z\}$ so that $n = 3$. The structure of 2^Ω is shown in a three dimensional boolean hypercube in Fig. 1, with the subsets of Ω related by inclusion relations. The example random set $\mathcal{S} = \{\langle\{x,z\}, .3\rangle, \langle\{y,z\}, .2\rangle, \langle\{x\}, .5\rangle\}$ is shown on the left of Fig. 2, with the Bel and Pl measures on the center and right, where zero values are not shown. For example, we have $\text{Bel}(\{z\}) = 0, \text{Pl}(\{x,z\}) = .5 = \rho(z) = .5$.

3. Probability and Possibility

When $\forall A_j \in \mathcal{F}(\mathcal{S}), |A_j| = 1$ then $\mathcal{S}$ is called specific, and $\text{Pr} := \text{Bel} = \text{Pl}$ is a probability measure. Pr is decomposable for $\sqcup = +_b$, where $x +_b y := (x + y) \wedge 1$ and $\wedge$ is the minimum operator. Normalization of Pr implies that $+_b = +$ here, so that

$$\text{Pr}(A \cup B) + \text{Pr}(A \cap B) = \text{Pr}(A) + \text{Pr}(B)$$

$$\text{Pr}(A \cup B) = \text{Pr}(A) + \text{Pr}(B) - \text{Pr}(A \cap B)$$

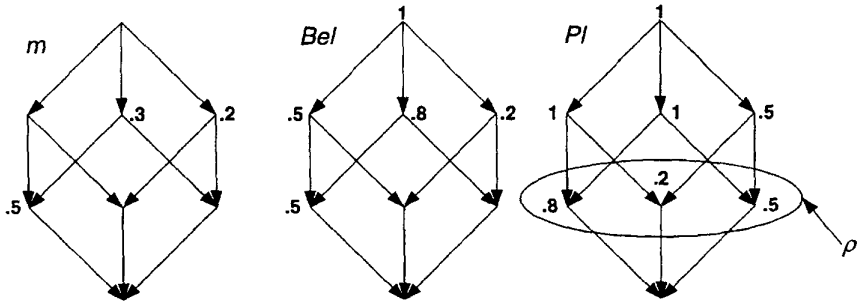

Figure 2. (Left) An example random set. (Center) Bel. *(Right)* Pl *and* ρ.

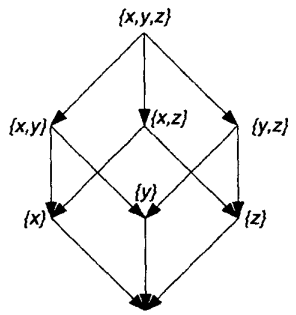

Figure 1. The structure of 2^{Ω}.

and $p := \rho$ becomes a probability distribution with $\Pr(A) = \sum_{\omega_i \in A} p(\omega_i)$.

When, for an arbitrary permutation of the j, and letting $A_0 = \emptyset$, $\forall A_j, A_{j-1} \subseteq A_j$, then $\mathcal{S}$ is called consonant and $\Pi := \mathrm{Pl}$ is a possibility measure. Π is decomposable for the maximum operator $\vee$, so that $\Pi(A \cup B) \vee \Pi(A \cap B) = \Pi(A) \vee \Pr(B)$, and from the monotonicity of ν and the fact that $A \cap B \subseteq A \cup B, \Pi(A \cup B) = \Pi(A) \vee \Pi(B)$. $\pi := \rho$ is now a possibility distribution with $\Pi(A) = \bigvee_{\omega_i \in A} \pi_(\omega_i)$. Further, we have

$$m(A_i) = \pi_i - \pi_{i+1}, \tag{4}$$

where A_i is that A_j such that $A_j = \{\omega_1, \omega_2, \ldots, \omega_i\}$, and $\pi_{n+1} := 0$ by convention.

4. Consistent Random Sets and Their Consonant Approximations

Other fuzzy measures available on random sets are decomposable, but it is strongly suggested [4] that possibility and probability are the only measures for which not only are the *measure* values $\mathrm{Pl}(A)$ determined by the singletion measure values $\rho_i = \mathrm{Pl}(\{\omega_i\})$, but so also is the identity of the *focal elements* A_j from the universe elements ω_i.

Therefore, the availability of probabilistic or possibilistic representations of random sets is a very valuable thing to do. However, the constraints of specificity or consonance are severe, rarely occurring in a pure form in generalized measurement situations. Thus we wish to identify where reasonable approximations which produce probability and possibility measures might be available, and at what cost in terms of accuracy and computation.

When $\mathbf{C}(\mathcal{S}) \neq \emptyset$, then $\mathcal{S}$ is called consistent. Of course, all consonant random sets are consistent, with $\mathbf{C}(\mathcal{S}) =$ that A_j such that $|A_j|$ is minimal. Consistent random sets have the following useful property:

Theorem 2 [2] A random set $\mathcal{S}$ is consistent if and only if $\bigvee_{\omega_i \in \Omega} \rho^{\mathcal{S}}(\omega_i) = 1$.

So given a consistent random set $\mathcal{S}$, its plausibility measure Pl is not a possibility measure, but its trace $\rho^{\mathcal{S}}$ *is* a possibility distribution, which we identify as $\pi^* := \rho^{\mathcal{S}}$. Since, in turn, every possibility distribution determines a unique possibility measure, it is possible to construct a new possibility measure Π^* based on $\rho^{\mathcal{S}}$ from $\forall A \subseteq \Omega, \Pi^*(A) := \bigvee_{\omega_i \in A} \pi^*(\omega_i)$, and from (4) a new evidence function m^* and consonant random set $\mathcal{S}^*$. Dubois and Prade have shown:

Theorem 3 [1] If $\mathcal{S}$ is a consistent random set, then $\mathcal{S}^* \subseteq \mathcal{S}$.

While we are considering general finite random sets, we have a special interest in random intervals defined as statistical collections of half-open intervals of $\mathbb{R}$ [5]. These structures are common in uncertain measurement problems. Note that while random intervals are defined on a continuous space, they are finite collections of intervals, and thus treatable as

finite random sets. An example is shown in Fig. 3, including the resulting possibilistic histogram.

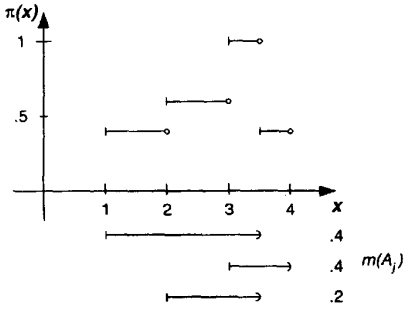

Figure 3. (Bottom) A consistent random interval. (Top) Its possibilistic histogram.

We now wish to characterize finite consistent random sets in general. For simplicity, consider a general consistent random set $\mathcal{S}$ which is complete in the sense that there is a unique singleton focus, and that there is positive evidentiary value for all subsets containing the focus:

$$\exists! \omega^* \in \Omega, \mathbf{C}(\mathcal{S}) = \{\omega^*\},$$

$$\mathcal{F}(\mathcal{S}) = \{A \subseteq \Omega | \omega^* \in A\}.$$

Then $\mathcal{F}(\mathcal{S})$ is the $n-1$-dimensional simplex dominated by ω^*. Thus $N = 2^{n-1}$, and $\mathbf{H}^{\mathbf{C}} := \mathbf{H}$ is the $n \times 2^{n-1}$ matrix shown in Fig. 4, choosing the permutation of the ω_i so that

$$\pi_i^* \geq \pi_{i+1}^*. \tag{5}$$

Thus from (3) we have

$$\vec{\pi}^{*T} = \mathbf{H}^{\mathbf{C}} \vec{m}^T, \tag{6}$$

where now $\vec{m}$ is a 2^{n-1} vector.

Given π^* characterized in this way, we now wish to construct $\mathcal{S}^*$. From (3), (4), and (6) we have

$$\begin{aligned} m_i^* &= \pi_i^* - \pi_{i+1}^* \\ &= \mathbf{H}^{\mathbf{C}}(i)\vec{m}^T - \mathbf{H}^{\mathbf{C}}(i+1)\vec{m}^T \end{aligned}$$

where $\mathbf{H}^{\mathbf{C}}(n+1)$ is the zero vector $\langle 0, 0, \ldots, 0 \rangle$. We can thus construct a new $n \times 2^{n-1}$ matrix $\mathbf{H}'^{\mathbf{C}}$ such that

$$\vec{m}_n^{*T} = \mathbf{H}'^{\mathbf{C}}_{n \times 2^{n-1}} \vec{m}^T_{2^{n-1}}, \tag{7}$$

where $\vec{m}^*$ is now an n, not a 2^{n-1}, vector, and the subscripts indicate matrix dimensions. We do this by simply letting

$$\mathbf{H}'^{\mathbf{C}}(i) := \mathbf{H}^{\mathbf{C}}(i) - \mathbf{H}^{\mathbf{C}}(i+1).$$

The results are shown in Fig. 5.

5. Towards Measures of Distortion

We now wish to consider the distortion introduced to a consistent random set $\mathcal{S}$ by its consonant approximation $\mathcal{S}^*$, and therefore the difference between $\mathrm{Pl}(A)$ and $\Pi^*(A)$.

5.1. Towards a Measure

First, from Thm. (3) we know that $\forall A \subseteq \Omega, \Pi^*(A) \leq \mathrm{Pl}(A)$. Therefore an appropriate measure for distortion could be something like

$$D(\mathcal{S}) := \sum_{A \subseteq \Omega} \mathrm{Pl}(A) - \Pi^*(A)$$

Since $\mathrm{Pl}(A) - \Pi^*(A) \in [0,1]$, therefore $0 \leq D(\mathcal{S}) \leq 2^{n-1}$, and so it might be wise to consider $\log_2 {}_2(D(\mathcal{S})) \leq n+1$. We have yet to consider the circumstances under which $D(\mathcal{S}) = 1$ so that $\log_2(D(\mathcal{S})) = 0$ is a critical point.

But whichever measure might eventually be used, we need to characterize $\mathrm{Pl}(A) - \Pi^*(A)$ better. We do so in our linear algebraic formulation as follows. First, since $\mathcal{S}$ is consistent and $\mathcal{S}^*$ consonant, we need to define structures corresponding to the matrix $\mathbf{G}^{\cap}$ restricted to those columns present. Thus define $\mathbf{G}^{\cap \mathcal{S}}$ as the matrix constructed by taking those columnns of $\mathbf{G}^{\cap}$ corresponding to the 2^{n-1} focal elements of $\mathcal{S}$, and correspondingly $\mathbf{G}^{\cap *}$ from those corresponding to the n focal elements of $\mathcal{S}^*$. Then, using the subscripts on some elements to indicate their matrix dimensions, we have from (7)

$$\begin{aligned} \vec{\mathrm{Pl}}^T_{2^n-1} &= \mathbf{G}^{\cap \mathcal{S}}_{2^n-1 \times 2^{n-1}} \vec{m}^T_{2^{n-1}} \\ \vec{\Pi}^{*T}_{2^n-1} &= \mathbf{G}^{\cap *}_{2^n-1 \times n} \vec{m}_n^{*T} \\ &= \mathbf{G}^{\cap *}_{2^n-1 \times n} \mathbf{H}'^{\mathbf{C}}_{n \times 2^{n-1}} \vec{m}^T_{2^{n-1}} \\ (\vec{\mathrm{Pl}} - \vec{\Pi}^*)^T_{2^n-1} &= (\mathbf{G}^{\cap \mathcal{S}} - \mathbf{G}^{\cap *}\mathbf{H}'^{\mathbf{C}})\vec{m}^T. \end{aligned}$$

5.2 An Example

The general case for $n = 3$ combined with a numerical example is shown in Fig. 6. The top of the figure shows m and Pl for a consistent random set on $\Omega = \{x, y, z\}$, where the ω_i have been ordered according to (5). Then we have

$$\begin{aligned} \mathcal{F}(\mathcal{S}) &= \{\{x\}, \{x,y\}, \{x,z\}, \{x,y,z\}\} \\ \mathcal{F}(\mathcal{S}^*) &= \{\{x\}, \{x,y\}, \{x,y,z\}\} \\ \vec{m} &= \langle .1, .3, .2, .4 \rangle \end{aligned}$$

$$\mathbf{H}^C = \begin{array}{c} \\ \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \\ \vdots \\ \omega_n \end{array} \begin{array}{c} \begin{array}{cccccccccccc} m_1 & m_2 & m_3 & m_4 & m_5 & m_6 & m_7 & m_8 & \dots & m_{2^{n-1}}-2 & m_{2^{n-1}}-1 & m_{2^{n-1}} \end{array} \\ \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & \dots & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & \dots & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & \dots & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & \dots & 1 & 1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 1 & 1 & 1 \end{pmatrix} \end{array}$$

Figure 4. $\mathbf{H}^C$ *for a consistent randomset.*

$$\mathbf{H}'^C = \begin{array}{c} \\ \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \\ \vdots \\ \omega_n \end{array} \begin{array}{c} \begin{array}{cccccccccccc} m_1 & m_2 & m_3 & m_4 & m_5 & m_6 & m_7 & m_8 & \dots & m_{2^{n-1}}-2 & m_{2^{n-1}}-1 & m_{2^{n-1}} \end{array} \\ \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & \dots & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 & -1 & 0 & \dots & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & \dots & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 1 & 1 & 1 \end{pmatrix} \end{array}$$

Figure 5. $\mathbf{H}'^C$ *for a consistent randomset.*

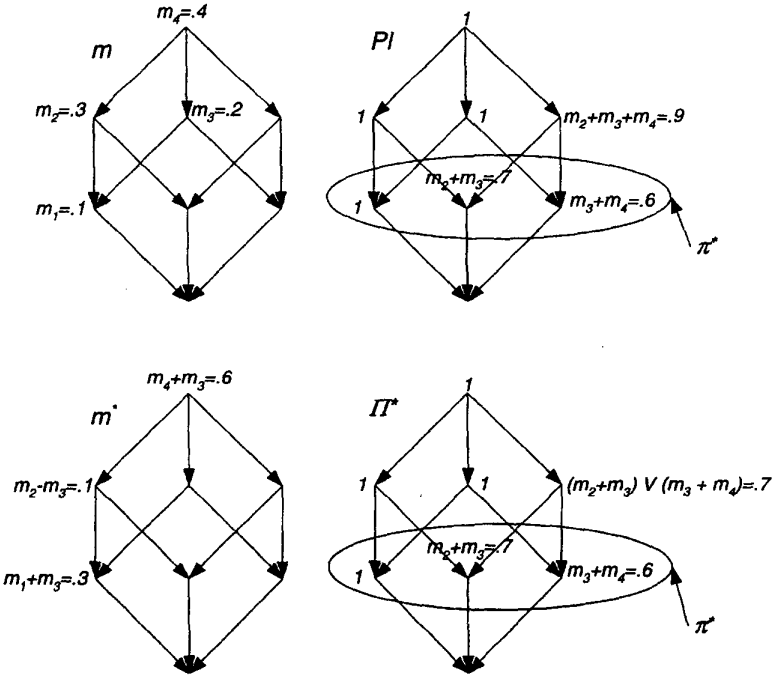

Figure 6. (Top) A consistent random set, m *and* Pl. *(Bottom left) The approximation* m^*. *(Bottom right) The approximation* Π^*.

$$\mathbf{H}^{\mathbf{C}} = \begin{array}{c} \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{array} \begin{array}{c} \begin{array}{cccc} m_1 & m_2 & m_3 & m_4 \end{array} \\ \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \end{array}$$

$$\mathbf{H}'^{\mathbf{C}} = \begin{array}{c} \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{array} \begin{array}{c} \begin{array}{cccc} m_1 & m_2 & m_3 & m_4 \end{array} \\ \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \end{array}$$

$$\mathbf{G}^{\cap S} = \begin{array}{c} \\ \{x\} \\ \{y\} \\ \{z\} \\ \{x,y\} \\ \{x,z\} \\ \{y,z\} \\ \{x,y,z\} \end{array} \begin{array}{c} \begin{array}{cccc} \{x\} & \{x,y\} & \{x,z\} & \{x,y,z\} \end{array} \\ \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \end{array}$$

$$\mathbf{G}^{\cap *} = \begin{array}{c} \\ \{x\} \\ \{y\} \\ \{z\} \\ \{x,y\} \\ \{x,z\} \\ \{y,z\} \\ \{x,y,z\} \end{array} \begin{array}{c} \begin{array}{ccc} \{x\} & \{x,y\} & \{x,y,z\} \end{array} \\ \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \end{array}$$

$$\begin{aligned} \vec{\mathrm{Pl}} - \vec{\Pi}^* &= (\mathbf{G}^{\cap S} - \mathbf{G}^{\cap *}\mathbf{H}'^{\mathbf{C}})\vec{m}^T \\ &= \langle 0,0,0,0,0,.2,0 \rangle \end{aligned}$$

The final result was verified computationally, and verifies the results determined numerically from the alternate formulation

$$\Pi^*(A) = \bigvee_{\omega_i \in A} \pi_i^* = \bigvee_{\omega_i \in A} \mathbf{H}^{\mathbf{C}}(i)\vec{m}^T,$$

and shown in the figure.

6 Conclusion

The work presented here is, if only for space reasons, incomplete. In particular, the properties of D need to be explored more completely. Moreover, the linear algebraic formulation used here needs to be placed in a more general combinatorical context, involving Möbius [9, 10] and/or Fourrier transformations.

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