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## A POSSIBILISTIC APPROACH TO QUALITATIVE MODEL-BASED DIAGNOSIS

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**Abstract**—The potential for the use of possibility theory in the qualitative model-based diagnosis of spacecraft systems is described. The first section of the paper briefly introduces the Model-Based Diagnostic approach to spacecraft fault diagnosis, Qualitative Modeling methodologies, and the concept of possibilistic modeling in the context of Generalized Information Theory. Then the necessary conditions for the applicability of possibilistic methods to qualitative model-based diagnosis, and a number of potential directions for such an application, are described.

### INTRODUCTION

Possibility theory is being developed as an alternative to traditional theories of uncertainty. While possibility is logically independent of probability theory, they are related: both arise in Dempster-Shafer evidence theory as fuzzy measures defined on random sets; and their distributions are fuzzy sets. Together these fields compose the new field of Generalized Information Theory (GIT).

Possibility distributions are built from a set of partially overlapping intervals, resting in non-additive, possibilistic weights on a set of alternatives. Possibilistic processes generalize both interval arithmetic methods and nondeterministic processes. Possibilistic models provide a Qualitative Modeling (QM) method which is a hybrid of interval and uncertainty distribution methods.

Qualitative methods are appropriate for modeling complex systems, such as spacecraft, where the interaction among the large number of parts under varying environmental conditions results in the possibility of unpredictable behavior and long-term departure from established steady-state domains. These methods inherently require loose representations of uncertainty, and typically involve interval analysis. Therefore it is hypothesized that possibilistic methods may be useful for qualitative model-based diagnosis and trend analysis of spacecraft systems.

### MODEL-BASED DIAGNOSIS

The model-based approach to systems diagnosis (MBD) (Hamscher, Console, & Kleer, 1992) is based on the premise that knowledge about the internal structure of

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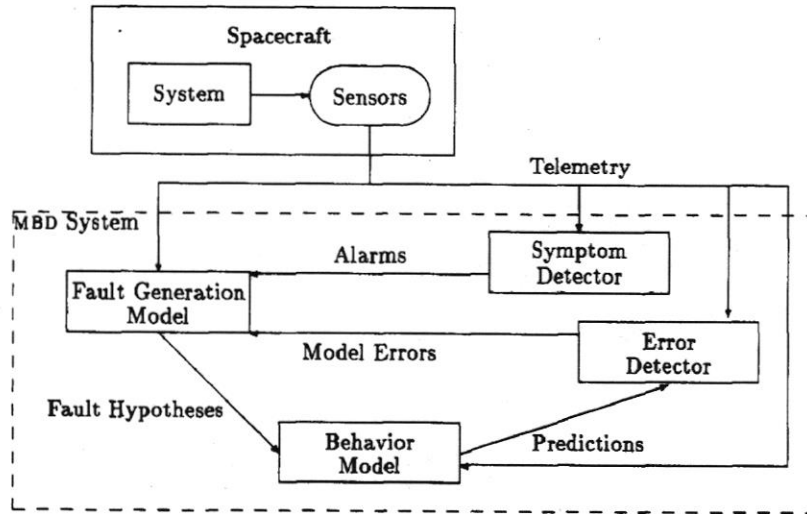


Figure 1. A typical model-based diagnostic system.

system can be useful in diagnosing its failure. In MBD, a software model of the system generates and tests various failure hypotheses when given inputs from the real system.

A typical MBD approach (derived from some of the standard literature [David Hamscher, 1992; Dvorak & Kuipers, 1992; Hall, Schuetzle, & La Vallee, 1992]) for diagnosing a spacecraft (here described as an internal system whose sensor measurements output to a telemetry stream) is shown in Figure 1. An alarm is a report that some observed system attributes have departed from nominal conditions and entered error conditions, usually by exceeding some threshold values. A prediction is a report that some system attributes should be in certain states. A fault hypothesis is a list of system components that may have failed. An error is a report of a discrepancy between predicted and measured system attribute states.

The overall MBD system then involves two distinct spacecraft models. The Fault Generation Model (FGM) takes inputs from telemetry, alarms, and errors, and either produces new, or modifies existing, fault hypotheses. The behavior model takes inputs from telemetry and fault hypotheses, and outputs predictions. These are then correlated against telemetry to produce errors. The fault hypotheses act to modify the behavior model so that it predicts system behavior as if the hypothetical system components had actually failed.

Both models can be difficult to construct, typically requiring delicate engineering tradeoffs among accuracy, precision, and tractability. But the FGM, as the heart of the MBD approach, is particularly complex and involved. The FGM could be, for example, an inversion of the behavior model (as for Dvorak and Kuipers [1992]) or a decision tree (as for Shen and Leitch [1991]). Through backward reasoning, a variety of subsets of components, failure of any of which is consistent with the given telemetry and alarms, can be identified.

Filtering is the process by which error output is used to prune the set of fault hypotheses. If the prediction of the behavior model as modified by a particular fault hypothesis produces errors, then that fault hypothesis is not retained. As the system is monitored over time, further observations narrow the class of viable fault hypotheses.

Achieving the null set indicates model insufficiency. But if the overall MBD system stabilizes to a non-empty set of fault-hypotheses, then these are advanced as possible causes of the failure.

## QUALITATIVE MODELING

Qualitative modeling, a branch of Systems Science usually considered to be a part of the field of Artificial Intelligence, can be broadly described as the attempt to model systems at deliberately high levels of abstraction. Of course, this approach produces models that are less precise than they might be, but with the tradeoff of potentially greater tractability and accuracy (the less you say, the better your chance of being right). QM methods can be useful when there is only a poor model of the original system, or when there are missing or incomplete data. This can happen when systems are incompletely specified, when they have parameters or states that aren't always known with certainty, or when complexity makes detailed prediction difficult.<sup>1</sup> There are a variety of broad approaches within QM, which can be known as naive physical qualitative physics, qualitative simulation, qualitative reasoning, qualitative dynamics etc. There are also a number of specific methods, including bond graphs, causal loop modeling, natural language modeling, "lumped" state space models and inductive approaches.

In this paper, the QM methods of most interest are those that use uncertainty distributions on state variables, or mixed interval- and point-valued dynamical systems. In models using uncertainty distribution methods, the uncertainty about some attribute is represented mathematically by weights on all possible values. The set of weights, as a distribution, acts as a meta-state in the space of all such distributions and functional equations relating these meta-states produce predictions about the meta-state distribution at future times. Models using probability distributions are familiar as Markov processes and other kinds of stochastic models, and these have correlated in possibility theory (see "Possibilistic Models" below). These methods are actually semi-qualitative, because the numerical representation of the distribution acts as a quantitative component.

In an interval-valued dynamical modeling system like QSIM (Kuipers, 1989), a precise point-valued dynamical system of differential or difference equations is replaced by a homomorphic interval-valued process. Typically, qualitative variables are identified within certain intervals, some relatively unconstrained (for example  $[0, \infty)$ ), and some constrained by "landmark" values (for example  $x \in [x_{\min}, x_{\max}]$  where  $x_{\min}$  and  $x_{\max}$  may or may not be known).

Qualitative variables are then generally related in three ways:

- Functional: For example, if  $y = M^-(x)$  then  $y$  is a monotonically decreasing function of  $x$ , so that if  $x \in [0, x_{\max}]$  then  $y \in (-\infty, 0]$  or  $y \in (M^-(x_{\max}), 0]$ .
- Arithmetic: Standard mathematical operations can also be represented qualitatively, for example if  $x \in [0, x_{\max}]$  and  $y \in (-\infty, 0]$ , then  $xy \in (-\infty, 0]$ , but  $x + y$  is unknown. These interval arithmetic methods (Moore, 1979) are closely related to possibility theory (see "Possibility Theory and Interval Analysis" below).
- Dynamic: Change of state is represented by qualitative magnitude and direction.

<sup>1</sup>For more information about QM in general, see the anthologies edited by Bobrow (1985), and Fishwick and Luker (1990), and survey articles by Fishwick (1989) and Guariso, Rizzoli and Werthner (1992).

and qualitative differential relations link directions with magnitudes. For example, given  $y = dx/dt$ , then

$$x \text{ increasing} \rightarrow y \in (0, \infty), \quad x \text{ decreasing} \rightarrow y \in (-\infty, 0).$$

Of course, determinative results may not be available in such qualitative models. For example, we saw above that  $x + y$  could be any value in  $(-\infty, \infty)$ . Similarly, the existence of potentially unknown landmark values leads to uncertainty as to whether a landmark has been crossed. To account for both possibilities, two alternatives must be considered independently, resulting in a branching nondeterminism. Therefore, in general, QM systems have a tree of possible system behaviors, and external factors (heuristics or other constraints) may be required to prune that tree.

QM has been applied to MBD to produce qualitative model-based diagnostic systems. For example, in the approach of Dvorak and Kuipers (1992), model predictions are intervals of possible system state values. Stochastic methods, for example Bayesian networks (Gertler & Anderson, 1992) and Markov processes (Grossman & Werth, 1993), have been used extensively in MBD applications. And recently, Shen and Leifer (1991, 1993) have advanced the FuSIm method for qualitative MBD which uses fuzzy arithmetic (see "Possibility Theory and Interval Analysis" below).

### POSSIBILITY THEORY

Possibility theory was originally developed in the context of fuzzy systems (Zadeh, 1978), and was thus related to the kinds of cognitive modeling that fuzzy sets are usually used for. More recently, possibility theory is being developed as a new form of mathematical information theory complementing probability theory in the context of GIT. The author is developing mathematical possibility theory on the basis of consistent random sets, and is also developing an empirical semantics for possibility theory including possibilistic measurement procedures and applications to the modeling of physical systems.

#### *Possibilistic mathematics in GIT*

Table 1 summarizes the primary formulae of probability and possibility theory in the context of GIT and random set theory. The mathematical basis of possibility theory in the context of GIT can only be briefly introduced here.<sup>2</sup>

Given a finite universe  $\Omega := \{\omega_i\}, 1 \leq i \leq n$ , the function  $m: 2^\Omega \rightarrow [0, 1]$  is an evidence function when  $m(\emptyset) = 0$  and  $\sum_{A \subseteq \Omega} m(A) = 1$ .  $m$  effectively acts as a probability distribution on the subsets of  $\Omega$ , and so a random set generated from an evidence function is denoted

$$\mathcal{S} := \{ \langle A_j, m_j \rangle : m_j > 0 \},$$

$$A_j \subseteq \Omega, \quad m_j := m(A_j), \quad 1 \leq j \leq N := |\mathcal{S}| \leq 2^n - 1,$$

where  $\langle \cdot \rangle$  is a vector and each  $A_j$  is a focal element. Denote the focal set as  $\mathcal{F} := \{ \langle A_j, m_j \rangle : m_j > 0 \}$  with core and support

<sup>2</sup>See Dubois and Prade (1988), Joslyn (1994c), Klir (1993), and Klir and Folger (1987) for more information.

$$C(\mathcal{F}) := \bigcap_{A_j \in \mathcal{F}} A_j, \quad U(\mathcal{F}) := \bigcup_{A_j \in \mathcal{F}} A_j$$

respectively.

Random sets were first developed in the context of stochastic geometry, dealt generally with random compact subsets of  $\mathbb{R}^n$  (Kendall, 1974; Matheron, 1975). It is only more recently that random sets are being integrated into the formalisms of G (Mahler, 1993; Nguyen, 1978).

In Dempster-Shafer evidence theory (Shafer, 1976)  $m$  is called a "basic probability assignment" and  $\mathcal{S}$  a "body of evidence". Given a random set or body of evidence, the plausibility and belief measures are defined on all  $A \subseteq \Omega$  as

$$Pl(A) := \sum_{A_j \cap A \neq \emptyset} m_j, \quad Bel(A) := \sum_{A_j \subseteq A} m_j$$

They establish respectively an upper and lower bound on  $Pr(A)$ . Since they are dual in that  $Bel(A) = 1 - Pl(\bar{A})$ , in general only plausibility will be considered below.

The plausibility assignment (otherwise known as the trace or one-point coverage function) of  $\mathcal{S}$  is  $\bar{Pl} = \langle Pl_i \rangle := \langle Pl(\{\omega_i\}) \rangle$ , where

$$Pl_i := \sum_{A_j \ni \omega_i} m_j, \quad (1)$$

and the summation is over all the  $A_j$  containing  $\omega_i$ .

Since each  $Pl_i \in [0,1]$ , therefore  $\bar{Pl}$  can be taken as a fuzzy subset of  $\Omega$ , where  $\bar{Pl}_i$  indicates the degree or extent to which  $\omega_i$  is included in the set. In turn, a given fuzzy set can be mapped to a class of one-point equivalent random sets on  $\Omega$  (Joslyn, 1994). This provides a key link within GIT between fuzzy theory and both random set and possibility theory (see "Possibility Theory and Fuzzy Theory" below).

Under certain conditions the evidence values  $m_j$  and the plausibility assignment values  $Pl_i$  are mutually determining. Then  $N \leq n$ , and  $\bar{Pl}$  is called a distribution of focal elements. When  $N = n$  (there are exactly as many focal elements as there are elements of the universe), then the indices  $j$  on the focal elements  $A_j$ , and the indices  $i$  on the universe elements  $\omega_i$ , are equivalent, and it may be useful to use one or the other interchangeably. This is the case in the two rightmost columns of Table 1.

When  $\forall A_j \in \mathcal{F}, |A_j| = 1$ , then  $\mathcal{S}$  is specific, and  $Pr(A) := Pl(A) = Bel(A)$  is an additive probability measure with probability distribution  $\bar{p} = \langle p_i \rangle := \bar{Pl}$  and additive normalization  $\sum_i p_i = 1$  and operator  $Pr(A) = \sum_{\omega_i \in A} p_i$ .

$\mathcal{S}$  is consonant ( $\mathcal{F}$  is a nest) when (without loss of generality for ordering, and let  $A_0 := \emptyset, A_{j-1} \subseteq A_j$ ). Now  $\Pi(A) := Pl(A)$  is a possibility measure, and  $\eta(A) := Bel(A)$  is a necessity measure. Since results for necessity are dual to those of possibility, only possibility will be discussed in the sequel.

As  $Pr$  is additive, so  $\Pi$  is "maximal":

$$\forall A, B \subseteq \Omega, \Pi(A \cup B) = \Pi(A) \vee \Pi(B),$$

where  $\vee$  is the maximum operator. As long as  $C(\mathcal{F}) \neq \emptyset$  (this is required if  $\mathcal{F}$  is a nest), then  $\bar{\pi} = \langle \pi_i \rangle := \bar{Pl}$  is a possibility distribution with maximal normalization.

Table 1. Summary of probability and possibility in GIT

		Distributions: $i \leftrightarrow j$	
	Random Set	Probability	Possibility
Focal Set	Any	Singletons: $A_i = \{\omega_i\}$	Nest: $A_i = \{\omega_1, \dots, \omega_i\}$
Structure	None	$\{\omega_j\} = A_i$ Partition	$\{\omega_j\} = A_i - A_{i-1}, A_0 := \emptyset$ Total order
Belief	$\text{Bel}(A) = \sum_{A_j \subseteq A} m_j$	$\text{Pr}(A) := \text{Bel}(A)$	$\eta(A) := \text{Bel}(A)$
Plausibility	$\text{Pl}(A) = \sum_{A_j \cap A \neq \emptyset} m_j$	$\text{Pr}(A) := \text{Pl}(A)$	$\Pi(A) := \text{Pl}(A)$
Relation	$\text{Bel}(A) = 1 - \text{Pl}(\bar{A})$	$\text{Bel}(A) = \text{Pl}(A) = \text{Pr}(A)$	$\eta(A) = 1 - \Pi(\bar{A})$
Distribution	$\text{Pl}_i = \sum_{A_j \supseteq A_i} m_j$	$p_i := \text{Pl}_i = m_i$  $m_i = p_i$	$\pi_i := \text{Pl}_i = \sum_{j=i}^n m_j$  $m_i = \pi_i - \pi_{i+1}, \pi_{n+1} := 0$
Measure		$\text{Pr}(A \cup B) = \text{Pr}(A) + \text{Pr}(B) - \text{Pr}(A \cap B)$	$\Pi(A \cup B) = \Pi(A) \vee \Pi(B)$
Normalization		$\sum_i p_i = 1$	$\bigvee_i \pi_i = 1$

Nonspecificity	$\sum_i m_i \log_2  A_i $	0	$\sum_{i=2}^n \pi_i \log_2 \left  \frac{i}{i-1} \right  = \sum_{i=1}^n (\pi_i - \pi_{i+1}) \log_2(i)$
Strife	$-\sum_i m_i \log_2 \left[ \sum_{k=1}^n m_k \frac{ A_i \cap A_k }{ A_i } \right]$	$-\sum_i p_i \log_2(p_i)$	$\sum_{i=2}^n \pi_i - \pi_{i+1} \log_2 \left[ \frac{i^2}{\sum_{j=1}^i \pi_j} \right] < .892$
Semiring	$(\oplus, \otimes)$	$(+, \times)$	$(\vee, \sqcap)$
Marginals		$\rho(x) = \sum_y \rho(x,y)$	$\pi(x) = \bigvee_y \pi(x,y)$
Conditionals		$\rho(x,y) = \rho(x y) \times \rho(y)$	$\pi(x,y) = \pi(x \sqcap y) \sqcap \pi(y)$
		$\rho(x y) = \rho(x,y)/\rho(y)$	$\pi(x \sqcap y) \in [\pi(x,y), 1]$
		$\forall y, \sum_x \rho(x y) = 1$	$\forall y, \bigvee_x \pi(x \sqcap y) = 1$
		$\mathbf{P} := [\rho(x y)]$	$\mathbf{\Pi} := [\pi(x \sqcap y)]$
Process		$\rho' = \rho \cdot \mathbf{P}$	$\pi' = \pi \circ \mathbf{\Pi}$
		$\rho'(x) = \sum_y \rho(y) \times \rho(x y)$	$\pi'(x) = \bigvee_y \pi(y) \sqcap \pi(x \sqcap y)$
Concepts		Division among distinct hypotheses Frequency Chance Likelihood	Coherence around certain hypotheses Capacity Ease of attainment Distance, similarity

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$\sum_i \pi_i = 1$  and operator  $\Pi(A) = \sum_{\omega_i \in A} \pi_i$ . Also define the core and support of the possibility distribution

$$\mathbf{C}(\pi) := \{\omega_i : \pi(\omega_i) = 1\} = \mathbf{C}(\mathcal{S}), \quad \mathbf{U}(\pi) := \{\omega_i : \pi(\omega_i) > 0\} = \mathbf{U}(\mathcal{S})$$

Nonspecificity and strife are defined as two uncertainty measures defined on random sets

$$\mathbf{N}(\mathcal{S}) := \sum_j m_j \log_2 |A_j|$$

$$\mathbf{S}(\mathcal{S}) := - \sum_j m_j \log_2 \left[ \sum_{k=1}^n m_k \frac{|A_j \cap A_k|}{|A_j|} \right],$$

measuring respectively the possibilistic and probabilistic aspects of the uncertainty information, represented in the random set. They achieve the forms shown in Tab for the possibilistic and probabilistic special cases. Note that in the probabilistic case the uncertainty collapses to stochastic entropy, while in the possibilistic case, the st is bounded above by a small number.

Stochastic and possibilistic processes project a probability or possibility distribution forward in time according to a given set of conditional relationships. They are defined on their respective distributions when two operators  $\oplus$  and  $\otimes$  are available such that  $(\oplus, \otimes)$  form a semiring ( $\otimes$  distributes over  $\oplus$ ), and  $\oplus$  is the operator of the distribution. For probability,  $\oplus = +$ , and  $(+, \times)$  is the unique semiring.

For possibility,  $\oplus = \vee$ , and there are many semirings of the form  $(\vee, \square)$ , where  $\square$  is a triangular norm: a monotonic, associative, commutative operator with identity (Dubois & Prade, 1988).  $\wedge$  (the minimum operator) and  $\times$  are two of the most popular norms, as is  $0 \vee (x + y - 1)$ . Conditional possibility is not always unique depending on the norm used. The formulae for marginal, joint, and conditional probability and possibility (which is dependent on  $\square$ ) are then shown in the table, as are next state functions for stochastic and possibilistic processes (see next section).

### Possibilistic models

Probability and possibility almost never coincide. It is only when  $\exists! \omega_i, \mathcal{F} = \{\{\omega\}$  that

$$\vec{\mathbf{P}}_1 = \langle 0, 0, \dots, 1, \dots, 0, 0 \rangle,$$

so that  $\vec{\mathbf{P}}_1$  is both a probability and a possibility distribution. Semantically, probability and possibility theory are also related to very different concepts (Joslyn, 1993). Probability is inherently additive, and is thus concerned with the dispersal or division of knowledge over a set of distinct hypotheses, and so with concepts related to frequency.

But possibility is inherently *non-additive*. It is concerned with the *coherence* of knowledge *around* a set of certain hypotheses (the core  $\mathbf{C}(\pi)$ ), and thus with *order* concepts related to capacity. Where probability makes very strong constraints on representation of uncertainty (additivity), possibility makes only very *weak* constraints. The maximum relation is a very weak operator, and there is a choice of many norms to use, some of which are strong, and others of which, like  $\vee$ , are also weak.



## Qualitative model-based diagnosis

So possibilistic models are appropriate where stochastic concepts and methods are inappropriate, including situations where long-run frequencies are difficult if not impossible to obtain, or where small sample sizes prevail. This is true in reliability analysis, for example, where failures and system entry into non-nominal behavior domains are very rare; and trend-analysis, where even though observations are made over a long time, the state variables of concern change only very slowly, and new domains of behavior are only very rarely seen. In these cases, the weakness of the possibilistic representation is matched by the weak evidence available.

A general modeling relation is shown in Figure 2, where:  $t$  and  $t'$  are former and subsequent times;  $W = \{w\}$  is the set of states of the world;  $M = \{m\}$  is the set of states of the model;  $o: W \rightarrow M$  is the measurement function;  $r: W \rightarrow W$  is the movement of "reality"; and  $f: M \rightarrow M$  is the modeling prediction function. In a stochastic model,  $M = \{\bar{p}\}$ , where  $\bar{p}$  is a probability distribution of the state of the world, while in a possibilistic model,  $M = \{\bar{\pi}\}$ , where  $\bar{\pi}$  is a possibility distribution of the state of the world.

Reality is presumably given, or at least operates on its own without our help. So to make a valid model we are required to provide the measurement and prediction functions so that the diagram commutes. Measurement is used both to set the initial conditions of the model and to corroborate later measurements against the model state, while prediction is used to produce the future model state. So a possibilistic model naturally requires *possibilistic* measurement and prediction procedures.

Possibilistic measurement methods have been developed by the author (Joseph 1992, 1993c). The essential requirement is the collection of the frequency of occurrence of subsets or intervals that are partially overlapping. If the core of the observed intervals (their global intersection) is nonempty, then Equation 1 yields an empirical possibility distribution.

An example is shown in Figure 3. On the left, four observed intervals are shown. The bottom two occur with frequency 1/2, while each of the upper two have frequency 1/4. Together they determine an empirical random set. The step function on the right is the possibilistic histogram derived from Equation 1.

There are a variety of well-justified continuous approximations to a possibilistic histogram. Two examples are shown in the figure. The rising diagonal on the left is common to both. The two falling continuous curves on the right are distinct to each. The trapezoidal form marked  $\pi^*$  is one of the most commonly used continuous approximations, but it must be noted that this is only one possibility among many including smooth curves. This approach to possibilistic measurement generalizes to intervals and to the continuous case.

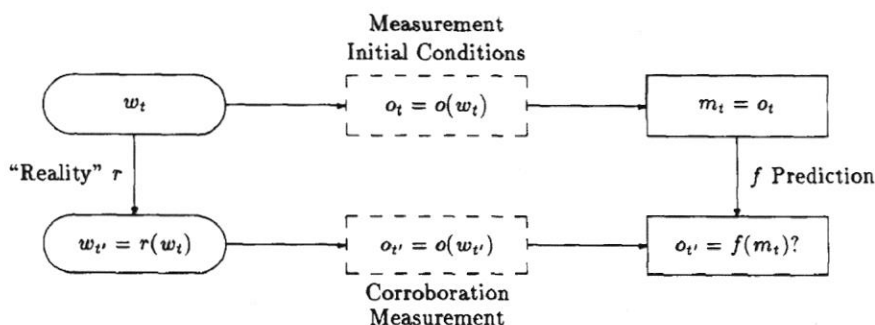


Figure 2. A general modeling relation.

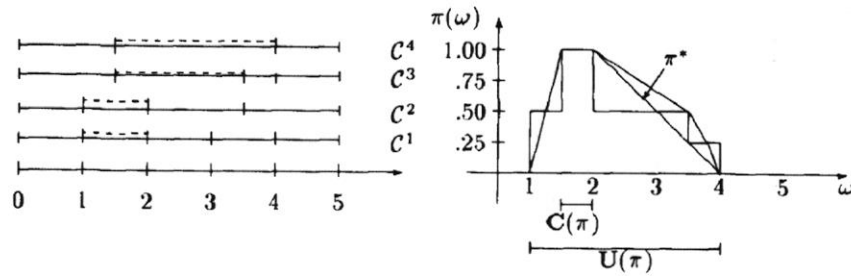


Figure 3. Four example observed intervals (left). The possibilistic histogram and two continuous approximations (right).

The core, here  $C(\bar{\pi}) = [1.5, 2]$ , being nonempty, is included in the support  $U(\bar{\pi}) [1, 4]$ . If the core were empty, then  $\forall_i P_i < 1$ , so that  $\bar{\pi}$  would not be a possibilistic distribution. In this case, possibilistic normalization procedures, which have also been developed by the author (Joslyn, 1993a), would be required.

Possibilistic prediction procedures can be based on the possibilistic processes introduced in the previous section. For example, the author has defined possibilistic automata as possibilistic Markov processes (Joslyn, 1994a) which generalize non-deterministic automata.

As an example of a possibilistic process, let  $\Omega = \{x, y, z\}$ . In Figure 4, each node is a state  $\omega_i$ . The arcs indicate state transitions, each of which is non-additively weighted with the conditional possibility of the transition. So, for example,  $\pi(x|y) = .8$ , the entire transition matrix is

$$\Pi = \begin{bmatrix} 0.0 & 0.8 & 0.0 \\ 1.0 & 0.0 & 0.0 \\ 0.2 & 1.0 & 1.0 \end{bmatrix}$$

Possibilistic transition normalization requires that each node have an out-arc label 1, so that each column of  $\Pi$  contains a 1. Nodes may have multiple out-arcs, however so that a unitary weight does not indicate a *necessary*, but rather a *completely possible* transition. For example, from state  $x$ , transition to  $z$  is .2 possible, but transition to  $y$  is also (completely) possible.  $z$  is an absorbing state, with self-transition the completely possible.

Now for  $t \geq 0$ , let  $\bar{\pi}^t = \langle \pi_i^t \rangle^T$  be the state possibility distribution vector, let  $\Gamma \in \Lambda$ , and assume that at time  $t = 0$  the system begins *definitely* in state  $x$ . Then  $\bar{\pi}^0 = \langle 1, 0, 0 \rangle^T$ , and  $\bar{\pi}^1$  is determined by

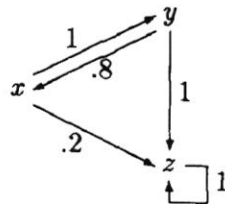


Figure 4. Weighted state transition diagram for a possibilistic process.

## Qualitative model-based diagnosis

Table 2. States of the Example Possibilistic Process.

$t$	0	1	2	3	4
$\bar{\pi}^t$	1.0	0.0	0.8	0.0	0.8
	0.0	1.0	0.0	0.8	0.0
	0.0	0.2	1.0	1.0	1.0
$N(\bar{\pi}^t)$	0.0	0.2	0.8	0.8	0.8

$$\bar{\pi}^1 = \bar{\pi}^0 \circ \Pi = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \circ \begin{bmatrix} 0.0 & 0.8 & 0.0 \\ 1.0 & 0.0 & 0.0 \\ 0.2 & 1.0 & 1.0 \end{bmatrix} = \begin{pmatrix} 0.0 \\ 1.0 \\ 0.2 \end{pmatrix},$$

where  $\circ$  is  $(\vee, \wedge)$  composition, which is verified by

$$\begin{aligned} \pi_1^1 &= (1 \wedge 0) \vee (0 \wedge .8) \vee (0 \wedge 0) = 0, \\ \pi_2^1 &= (1 \wedge 1) \vee (0 \wedge 0) \vee (0 \wedge 0) = 1, \\ \pi_3^1 &= (1 \wedge .2) \vee (0 \wedge 1) \vee (0 \wedge 1) = .2. \end{aligned}$$

The system is now *not* unequivocally in one state: Although it is completely possible that  $y$  is the system state,  $z$  is also .2 possible of being the system state, while  $x$  is impossible. Transition normalization guarantees that  $\bar{\pi}^t$  is always a maximally non-specific possibility distribution. Other future state vectors (and their nonspecificity values) are shown in Table 2.

When

$$\forall i, j, t, \quad \pi_i^t, \pi(\omega_i | \omega_j) \in \{0, 1\},$$

then "crisp" possibilistic processes, which are equivalent to nondeterministic processes, are recovered. In fact, it has been shown by the author (Joslyn, 1994a) that possibilistic processes are the valid generalizations of nondeterministic processes, whereas stochastic processes are not. The author has also developed possibilistic Monte Carlo methods (Joslyn, 1994c), which are required to select a final outcome based on the possibility distribution  $\bar{\pi}^t$  for a fixed  $t$ .

### POSSIBILITY THEORY AS A QUALITATIVE MODELING METHOD

Mathematical possibility, in both theory and applications, is still in the basic research phase, just out of its infancy. For example, the axiomatic basis for possibility theory and the properties of possibility distributions on continuous spaces are still to be defined. The semantics of possibility in physical systems has been considered only very few. But there are many reasons why it can be hoped, and even expected, that possibility theory can come to play an important role in QM in general, and in the application of QM to MBD in particular.

Hamscher, Console, and de Kleer (1992) have noticed some of the weaknesses of stochastic methods for MBD, "it is usually assumed that reliable failure statistics will be available, but this is in fact rare in practice. What is needed . . . is a way of work with likelihoods that could be specified ordinally rather than quantitatively" (p. 4). This is *exactly* what possibility theory provides, a non-additive, ordinal approach to QM that hybridizes interval-valued dynamics and uncertainty distribution methods

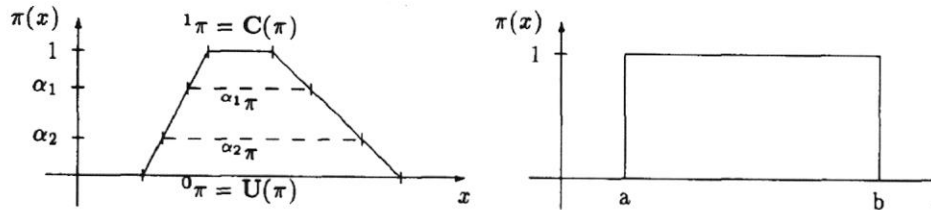


Figure 5. A possibility distribution as a collection of weighted intervals (left). The special case of a closed interval (right).

### Possibility theory and fuzzy theory

As mentioned above, possibility theory was originally developed by Zadeh (1978) in the context of fuzzy sets and fuzzy logic. For Zadeh, a possibility distribution simply was a fuzzy set by another name, and thus possibilistic information theory was strictly related to fuzzy information.

The view presented here, based in GIT, regards possibility theory as analogous to probability theory in the context of random sets. Here, possibilistic normalization ( $\sum \pi_i = 1$ ) becomes crucial, and both probability and possibility distributions are recognized as special cases of fuzzy sets with appropriate special normalization. In other words, while it is certainly true that a possibility distribution is a fuzzy set, in GIT there are many structures that are legitimately fuzzy sets, including probabilistic distributions. The author provides a full discussion elsewhere (Joslyn, 1994c).

### Possibility theory and interval analysis

Nevertheless, fuzzy theory and possibility theory do share a number of points in common. In particular,  $(\vee, \wedge)$  matrix composition is a common operation in fuzzy systems in general.

A fuzzy interval is a fuzzy set on  $\mathbb{R}$  that is also a convex possibility distribution, that

$$\forall x, y \in \mathbb{R}, \forall z \in [x, y], \pi(z) \geq \pi(x) \wedge \pi(y).$$

When  $\exists! x \in \mathbb{R}, \pi(x) = 1$ , then  $\pi$  is a fuzzy number. Fuzzy intervals and numbers are in fact maximally normalized possibility distributions, and all possibilistic histograms and their continuous approximations are fuzzy intervals (Joslyn, 1993c). As illustrated in Figure 5, under these conditions,  $\pi$  can be represented as a set of nested intervals weighted by their possibility values, where  ${}^0\pi := U(\pi)$  as a special case, and

$$\forall \alpha \in (0, 1], \alpha\pi := \{x \in \mathbb{R} : \pi(x) \geq \alpha\}, \alpha_1 \geq \alpha_2 \rightarrow \alpha_1\pi \subseteq \alpha_2\pi.$$

A standard interval  $[a, b] \subseteq \mathbb{R}$  is a special case of a fuzzy interval, where

$$\pi(x) = \begin{cases} 1, & a \leq x \leq b \\ 0, & x < a \text{ or } x > b \end{cases}, \quad \forall \alpha \in [0, 1], \quad \alpha\pi = [a, b].$$

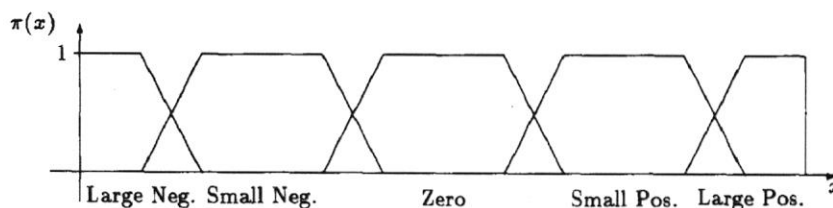


Figure 6. Traditional use of fuzzy intervals as linguistic variables.

Fuzzy arithmetic (Kaufmann & Gupta, 1985), where mathematical operations such as addition and multiplication are defined on fuzzy intervals and numbers, is a direct generalization of interval arithmetic (Moore, 1979). In this way, possibility theory can be used as a generalization of interval analysis, and QM methods that use fuzzy arithmetic are essentially possibilistic.

Fuzzy arithmetic is also very popular and common in applications, and has been used as a QM method both generally (Sugeno & Yasukawa, 1993) and for MBD (Fishwick, 1991; Shen & Leitch, 1993). Here the standard methods of fuzzy control systems are used, where a set of overlapping fuzzy intervals divide a quantity space into a few linguistic, qualitative values like "large positive" and "small negative" (Figure 6). These fuzzy sets are almost never *measured* properties of the system being modeled, and are instead heuristically specified by the system modeler. Thus they are essentially modeling the cognitive state of some human expert, rather than directly modeling the system in question.

This contrasts sharply with possibilistic processes and models. First, possibilistic models are cast strictly within the context of mathematical possibility theory specifically, rather than fuzzy theory generally. They, thus, always imply strict possibilistic normalization. Also, they are based on measurement, in the form of possibilistic histograms and their continuous approximations, of the attributes of the system being modeled.

### A POSSIBILISTIC APPROACH TO MBD

At both the general level and in some specific ways, there are areas of MBD for which it is appropriate to consider a possibilistic approach.

#### *Possibilistic symptom and error detection*

Symptom and error detectors in a typical MBD system simply compare the measured value against a crisp interval of nominal or predicted values (Dvorak & Kuipers, 1999). This is inadequate because the resulting cutoff from nominal to error condition is essentially arbitrary. It is natural to use a fuzzy interval to generalize this, measuring either prediction errors or fault symptoms as the possibilistic distance of the telemeasured value from the predicted or nominal system state respectively.

Consider a measured value  $x$  compared against an error fuzzy interval of the form shown in Figure 5. Such a possibility distribution could be the output of the behavior model, for instance, and would then serve as input to the error detector. Then  $\pi(x)$  is the strength of the error or alarm raised. When  $x \in C(\pi)$ , then  $\pi(x) = 1$  and there is no alarm. When  $x \notin U(\pi)$ , then  $\pi(x) = 0$  and the alarm is complete. In between, an intermediate alarm is raised.

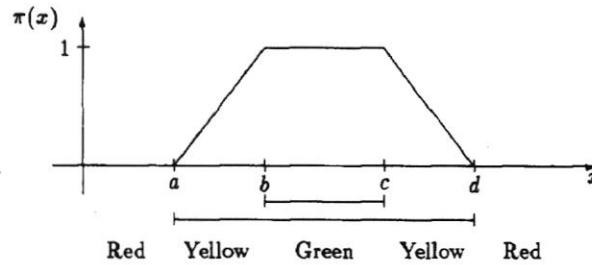


Figure 7. Degrees of alarms as a fuzzy interval.

A simple hypothetical example is illustrated in Figure 7. Here two crisp error thresholds  $[b, c] \subseteq [a, d]$  are provided, where  $[b, c]$  represents a conservative and  $[a, c]$  liberal judgment as to the nominal system state  $x$ . It is natural to let  $x \in [b, c]$  indicate the full "green" range, while  $x \in [a, b) \cup (c, d]$  indicates "yellow" status, grow "redder" as  $x \rightarrow a$  or  $x \rightarrow d$ . Finally,  $x \in (-\infty, a) \cup (d, \infty)$  indicates a full "r" status.

This is fully in keeping with the possibilistic approach, as indicated by the trapezoidal continuous approximation (shown in the figure) to the possibilistic histogram generated by the two crisp intervals. Here  $\pi(x)$  is the degree of "greenness" of system state, and  $\pi(x) \in (0, 1)$  indicates an intermediate degree of "yellow".

Even in situations where crisp thresholds are acceptable, they may be dynamic, varying as a result of changing system and environmental conditions. Doyle, Sell and Atkinson consider the situation of an earth-orbiting spacecraft as it proceeds through sunlight and shadow.

Impingement solar radiation changes the thermal profile of the spacecraft, as does the configuration of currently active and consequently, heat-generating subsystems on board. Thresholds on temperature sensors should be adjusted accordingly. A particular temperature value may be indicative of a problem when the spacecraft is in shadow or mostly inactive, but may be within acceptable limits when the spacecraft is in sunlight or many on-board systems are operating. (Doyle, Sellers & Atkinson, 1989: 1231)

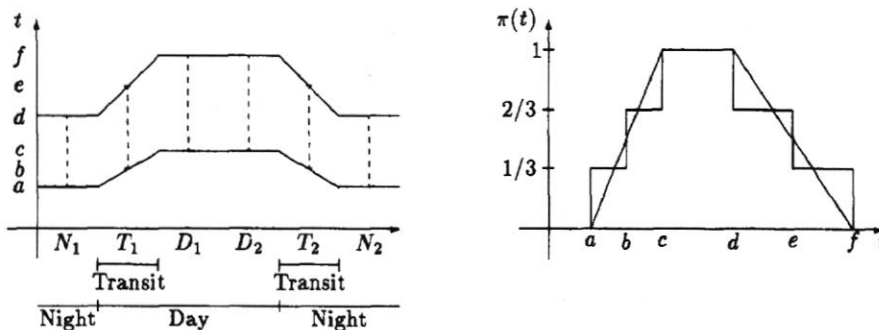


Figure 8. Variable critical range of a component through a day-night cycle (left). Its possibilistic histogram and a trapezoidal approximation (right).

This situation is shown in Figure 8. Assume a variable, say the temperature  $t$  of a given component, must be kept in a critical range as the spacecraft moves in and out of daylight. As it does so, the range shifts as shown in the upper left of the figure where the transition periods begin at a change in sunlight, and continue to their equilibrium. For simplicity, assume that that interval is sampled uniformly six times during the orbital day, twice each for daylight  $D_i$ , night  $N_i$ , and transition period for  $i \in \{1,2\}$ . The possibilistic histogram for the possibility  $\pi(t)$  of  $t$  holding a value at any given time and a trapezoidal approximation are shown on the right of the figure.

A combination of these two approaches is also possible, where instead of a critical interval changing over time, a whole possibility distribution as a generalized interval changes with time.

### *Sensor modeling*

Although the MBD system contains two models, the behavior model and the FGM, a whole, it is itself also a model of the spacecraft. As such, it is dependent on its input from measurement, and thus on the sensor output of the spacecraft. Thus, there are modeling issues in MBD concerning the sensors themselves.

When modeling complex systems, sensor data may be sparsely distributed, with missing observations, and sometimes very small sample sizes. As argued elsewhere by the author (Joslyn, 1993b), these are important conditions for the inapplicability of stochastic methods, and when they hold, possibilistic methods should be considered.

In this respect, there is strong support in the literature for the idea that possibilistic methods, as distinct from probability, has a role to play in QM. For example, Luo and Kay observe

When additional information from a sensor becomes available and the number of unknown propositions is large relative to the number of known propositions, an intuitively unsatisfying result of the Bayesian approach is that the probabilities of known propositions become unstable. (Luo & Kay, 1989)

While Durrant-Whyte's (1987) approach is statistical, he also notes, "a robot system uses notably diverse sensors, which often supply only sparse observations that cannot be modeled accurately." Dvorak and Kuipers (1992) make a similar observation in the context of model-based monitoring, "all measurements come from sensors, which can be expensive and/or unreliable and/or invasive. Monitoring is typically based on a small subset of the system parameters, with limited opportunity to probe other parameters."

### *Data fusion*

Possibilistic measurement, as outlined above, is predicated on the observation of sets or intervals that are partially overlapping. It is, therefore, imperative to consider the source of these intervals. But traditional measurement methods do not generally yield overlapping intervals. Rather, the purpose of designing a good sensor is to produce distinct outcomes, perhaps intervals with some uncertainty, but still disjoint, forming equivalence classes.

Random set theory in general and possibility theory in particular is significant when considering the problem of data fusion in MBD (Hackett & Shah, 1990; Luo & Kay, 1989). This is because overlapping intervals may result from the combination of dis-

from *different* instruments that measure the same system attribute, either directly or indirectly. This can be seen in Figure 3, where each  $\mathcal{C}^k$  is a different measuring device. While each  $\mathcal{C}^k$  produces disjoint intervals, any two intervals taken from two different  $\mathcal{C}^k$  may or may not overlap.

Hackett and Shah (1990) discuss data fusion in general, including indirect measurements, as well as the Dempster-Shafer (that is, random set) approach, "every sensor sensitive to a different property of the environment; in order to sense multiple properties, it is necessary to use multiple sensors. A system using multiple sensors that sense a single property can be used." Dubois, Lang, and Prade (1992) have considered the data fusion problem using possibilistic logic, as has Mahler (1993) using random sets.

**Indirect measurements.** First, consider the situation where measurements of a component are not made directly, but rather knowledge of the state of the component is obtained indirectly by inference from the outputs of sensors attached to other components. Doyle, Sellers, and Atkinson (1989) offer an example from jet aircraft: Engine thrust can be indicated by either low exhaust temperature, low turbine rotational speed, or both.

This situation is illustrated in Figure 9. Here component A is not monitored directly; its state can only be inferred from the sensors D and E, which monitor components B and C, and which, in turn, are causally connected to A. All of the intervals that can be reported by either D and E individually are distinct from each other and disjoint. However, since the knowledge of A provided by D and E is mediated by B and C, together they may indicate that A exists in two different, possibly overlapping, intervals. So as the amount of sensor "penetration" (sensor/component ratio) drops, standard measurement methods yielding frequency distributions may become less tenable, leaving only observations at the random set level.

**Redundant measurements.** Alternatively, a system component may be monitored redundantly by multiple instruments, as shown in Figure 10. If these sensors are identical, and identically calibrated, then the result will simply be as if there was a time-series of observations from a single instrument. But if they are mutually miscalibrated, either out of phase, or scale, or both, then the intervals reported from each instrument do not overlap.

If the sensors measure distinct modalities of a single component, for example pressure and temperature, then a process of registration (Hackett & Shah, 1990) is required.

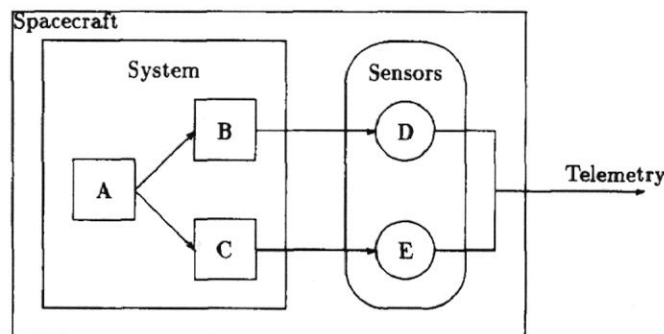


Figure 9. Indirect measurements of the state of a spacecraft component.



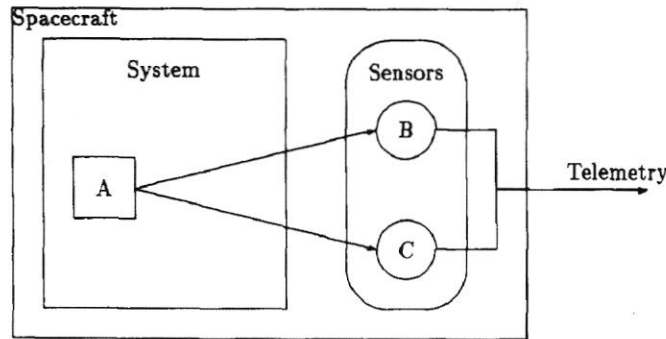


Figure 10. Redundant measurements of the state of a spacecraft component.

to derive a report from one in the modality of the other, or two new reports from each in a third modality. In any event, the argument here is very similar to the one above the case of indirect measurements, and possibly overlapping intervals may result.

### Sensor failure modeling

As mentioned above, in MBD data are not only combined from disparate sensors, they are also sometimes incomplete, degraded, or missing altogether. Even when standard (disjoint) observations are made, under these conditions there is the potential for application of GIT and possibility theory.

First, in GIT standard measurements are represented as singleton sets  $\{\omega_i\}$ , where each  $\omega_i \in \Omega$  may indicate a disjoint interval. In a Bayesian or stochastic approach sensor failure is represented by a uniform distribution over each of the  $\{\omega_i\}$ , again dividing our ignorance among a set of disjoint choices.

But in GIT, a missing observation is represented, more accurately, as an observation of the *entire* universe  $\Omega$ . While this does not result in a specifically possibilistic situation, neither does it result in a frequency or probability distribution. This is discussed more fully by the author elsewhere (Joslyn, 1994c).

For a simple example, assume that a system with three states  $\Omega = \{a, b, c\}$  is observed at ten uniformly distributed times, with  $a$  and  $c$  each seen twice,  $b$  seen three times, and three cases where the sensor made no report. These final three cases may be recorded as observations of  $\Omega$ , and the specific observations replaced with observations of the singleton sets  $\{a\}$ ,  $\{b\}$ , and  $\{c\}$  respectively. The overall empirical random set is then

$$S = \{ \langle \{a\}, 1/5 \rangle, \langle \{b\}, 3/10 \rangle, \langle \{c\}, 1/5 \rangle, \langle \Omega, 3/10 \rangle \}.$$

The plausibility assignment is then  $\tilde{P}I = \langle 1/2, 3/5, 1/2 \rangle$ , which is neither an additive probability distribution nor a maximal possibility distribution.

When sensor data are not missing, but rather degraded, compromised, or suspect some way, a confidence weighting on each sensor's output is naturally not additive; our confidence about the sensors is not divided among them, since all could be perfect or any number of them could be in any state of degradation. Instead, it is natural to represent this confidence as a possibility distribution (or general fuzzy set) on each sensor's output. Again, an observation in the core indicates complete confidence, while one outside the support indicates complete sensor failure.

Representation of a graduated degree of sensor failure allows a corresponding graduated degree of confidence in model predictions. The need for this has been noted by Fulton (1990), "when we detect a broken sensor, great difficulty arises if we continue diagnosing other failures, because typical rule-based systems do not degrade gracefully when sensors fail (because the mapping is dependent on a complete and accurate set of sensor data)."

### *Possibilistic models proper*

In the sequel, the term "system model" will refer to the FGM or the behavior model, generally. So finally, it is useful to consider possibilistic methods applied directly to the system models themselves, constructing them as possibilistic processes such as possibilistic automata, and not as fuzzy arithmetic systems as discussed above.

Input to these systems (telemetry, alarms, and errors) may or may not be *proper* possibility distributions, since both crisp (standard) intervals and point values are special cases of possibility distributions. But if they are, then it was discussed how they can be possibilistically weighted. A possibilistic FGM then would be responsible for producing as its output a set of fault hypotheses that are possibilistically weighted as input to the behavior model. This would, in turn, generate model prediction errors with possibilistic weights.

A system model that is a possibilistic automaton can also be cast as a possibilistic Markov process. As such, its key component is its transition matrix  $\Pi$ , essentially a vector of conditional possibility distributions as discussed above. Each conditional possibility distribution represents the possibility, for a given input, of transiting from one system state to another.

The semantics of this transition matrix in a system model is understood in terms of a subsystem-level model where the conditional possibilistic weight indicates a non-additive coupling or relatedness among subsystems. This could be, for example, the efficiency of the subsystem, as in the approach of Doyle, Sellers, and Atkinson (1988). Or, when considering the system model as a causal graph, as in the approach of Healey, Schuetzle, and La Vallee (1992), the weights indicate the degree of causal connectivity between subsystems.

Thus in the possibilistic approach a system model is essentially a possibilistic network, where nonadditive, possibilistic weights are placed on the arcs of a causal graph. The corresponding network appears similar to a Bayesian network, but the mathematics is possibilistic, not stochastic.

## CONCLUSION AND FUTURE DIRECTIONS

We have considered possibility theory as a qualitative modeling method in the context of GIT (probability theory, Dempster-Shafer evidence theory, random set theory, a fuzzy theory). We have also defined the key concepts of model-based diagnosis, and have considered, in the context of spacecraft diagnosis, the potential for the application of possibility theory to MBD in terms of symptom and error detection, diagnosis, fusion, sensor failure modeling, and nonadditive causal graphs.

It should be emphasized that this work is still in the basic research phase. Mathematical possibility theory is still being developed, and most of the key concepts in possibilistic modeling (for example, possibilistic measurement and automata) have only been defined in the past year or so. The application of possibility theory to the modeling

physical systems and the semantics of possibility in empirical contexts is being considered only by a few.

This work points to many future directions for research, including computer-based implementation of possibilistic models proposed by the author (Joslyn, 1994b), a continued exploration of the conditions for the applicability of possibilistic models to spacecraft systems.

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