Semantic Webs: A Cyberspatial Representational Form for Cybernetics

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Abstract

Mathematical systems theory suggests structures which are useful for representing cyberspatial and hypertextual systems. In particular, we introduce semantic webs based on acyclic multirelational systems as a model for typed link hypertexts. Such semantic webs are like semantic networks where concepts participate in a semantic category only in one direction. The resulting structures have indirect cyclicity in virtue of their multiple, overlapping loose hierarchies. Furthermore, they map easily into both existing and proposed hypertext systems, and resonate with many of the key ideas in Systems Science.

1 Introduction

As Cybernetics and Systems Science (which we here simply call Systems Science) move into their second half-century, the computer technologies that underlie them and finally allow their full expression are also coming of age. The continued exponential growth of computer technology is well known. As evolutionary theory suggests, such vast quantitative change is apt to result in the qualitative emergence of novel phenomena. In the first half of the 1990's this phenomena is the relatively instant globalization of telecommunications, and in particular the emergence of the Cyberspehere or Cyberspace in the form of the Internet and the World Wide Web (WWW).

This new technology, and the hypertextual forms which underlie it, have vast significance for Systems Science, and vice versa. They require new concepts and new representational forms, which are also both suggested by and suggestive for Systems Science. These ideas are being explored in the context of the PRINCIPIA CYBERNETICA Project [Heylighen *et al.*, 1995; Joslyn *et al.*, 1993][1], which reflexively applies cybernetic technologies and principles to their own development, and in particular is developing a WWW hypertext corpus for an evolutionary, systemic philosophy.

In this paper I will advance a cluster of simple ideas from mathematical systems theory which I believe point towards a useful and appropriate representation of cyberspatial structures. In particular, I define a "semantic web" based on acyclic multirelations as a model for typed link hypertextual corpi. The resulting systems have multiple, overlapping loose hierarchies with only indirect cyclicity. They map easily into both existing and proposed hypertext systems, and resonate with many of the key ideas in Systems Science.

2 Semantic Networks, Directed Graphs, and Hypertext

One point of departure, on which will not dwell at length here, is the formalism of mathematical systems theory [Klir, 1991], which is generally grounded in structures such as strings, networks, and trees in a discrete, Cartesian space. These are distinguished from the kinds of mathematical objects like curves and surfaces in continuous Euclidean spaces in which most of us have been trained.

2.1 Relational Systems

These structures are kinds of discrete relations, that is, subsets of some, perhaps complex Cartesian spaces. Many of them are best expressed in graph theoretical form as directed graphs, or node-arc diagrams: the nodes denote points in the space, and a link from one to another denotes an ordered pair in a binary relation, meaning that those nodes participate in the relation.[2] Mathematically, we would say the following.

Definition 1 (Binary Relational System) Given a set X, a **binary relational system** on X is a system $\mathcal{R} = \langle X, R \rangle$ where $R \subseteq X^2$.

As an example, let $X = \{a, b, c\}$ and

$$R = \{\langle a, b\rangle, \langle a, c\rangle, \langle b, a\rangle, \langle b, c\rangle, \langle c, c\rangle\} \subseteq X^2.$$

In addition to this algebraic form, $\mathcal{R}$ can also be represented in tabular form as an incidence matrix

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[1] http://pespmc1.vub.ac.be.

[2] It should also be noted that there are important relations here to category theory, which also has at its heart directed graph structures in the form of morphisms. Category theory has also been offered as a canonical language for systems theory [Rosen, 1985].

M. Now the set is written along two axes, and the presence of a pair $\langle x, y \rangle \in R$ is indicated by a check or a one in the cell at row x column y. For our example, this is

$$M = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \left(\begin{array}{ccc} & \surd & \surd \\ \surd & & \surd \\ & & \surd \end{array} \right) \end{array}$$

$\mathcal{R}$ can also be represented as a directed graph, or digraph, where now each element of the set is listed separately as a node, and a pair $\langle x, y \rangle$ is indicated by an arrow $x \to y$. For our example this is shown in Fig. 1.

2.2 Hypertext as a Binary Relational System

The mathematical structure of the binary relation, usually represented as a directed graph, is a fundamental representational form for both cyberspace and the hypertext on which it is currently, and probably into the future, based. Now the elements of X are some kind of hypertextual structure, dependent on the implementation. For example, in the HyperText Markup Language (HTML)[3] on the WWW nodes are web pages or `<a name=""></a>` anchors within a web page, and are denoted by distinct `URL`s. Then an ordered pair $\langle x, y \rangle \in R$ indicates a hyperlink from the `URL` x to the `URL` y.

As an example, consider the WWW page

`http://pespmc1.vub.ac.be/AUG1992nodes/VARIETY.html`,

a node concerning the concept "variety" in the PRINCIPIA CYBERNETICA Web. Denote this node as v. It has links to and from other nodes: the node for "distinction", denoted d, links to v, because the presence of a distinction creates variety; and v links to the node for "collection", denoted c, because we regard variety as a collection of distinctions; and finally v links to the node describing the works of Ross Ashby, denoted a, since Ashby is a noted author on variety.

Thus in the set of all WWW nodes X, we have $v, d, c, a \in X$. In terms of links, since d links to v, therefore $\langle d, v \rangle \in R$, and so on, so that $\{\langle d, v \rangle, \langle v, c \rangle, \langle v, a \rangle\} \subseteq R \subseteq X^2$. In this way, in principle the entire PRINCIPIA CYBERNETICA Web, indeed the entire World Wide Web, can be represented as one, huge, directed graph.

2.3 Typed Link Hypertext and Multirelational Systems

Of course, HTML and the WWW represent just one possible implementation of hypertext. And within the larger hypertext world there are other models (for example, the HYTIME system [Newcomb *et al.*, 1991]) which use a crucial feature not present in HTML: multiple kinds of link *types*.

In a typed link hypertext system, a given node a in a corpus might link to two nodes b and c, but in two different *ways*. Or even, a might link to b twice, once in one manner, and then again in another. In our example, the link from v to c is of a very different kind from the link from v to a or from d to v. It helps to imagine each link type uses a different color, for example.

Mathematically, we represent this as a set of distinct binary relations on the same set X, as in the following.

Definition 2 (Multirelational System) Given a set X, a **multirelational system** on X is a system $\mathcal{M} = \langle X, \{R_i\} \rangle$ where $R_i \subseteq X^2, 1 \le i \le n$.

$\mathcal{M}$ is represented in matrix form as a set of n incidence matrices M_i, one for each R_i. Graphically, each R_i is assigned a different arrow color or graphical pattern.

For an example, let $X = \{a, b, c, d\}$ and let $n = 2$, with

$$R_1 = \{\langle a, c \rangle, \langle a, d \rangle, \langle b, a \rangle, \langle b, d \rangle\},$$

$$R_2 = \{\langle a, b \rangle, \langle b, c \rangle, \langle d, c \rangle\}.$$

In matrix form, we have

$$M_1 = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \left(\begin{array}{cccc} & & \surd & \surd \\ \surd & & & \surd \\ & & & \\ & & & \end{array} \right) \end{array},$$

$$M_2 = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \left(\begin{array}{cccc} & \surd & & \\ & & \surd & \\ & & & \\ & & \surd & \end{array} \right) \end{array}.$$

Graphically this is shown in Fig. 2 on the left, where R_1 and R_2 are distinguished by dashed and solid arrows.

Note the following:

- Each of the R_i can be considered separately with X to form n binary relational systems $\mathcal{R}_i(\mathcal{M}) = \langle X, R_i \rangle$. These are shown in the center of Fig. 2.
- The R_i might overlap, so that $\exists i, i', R_i \cap R_{i'} \neq \emptyset$. Graphically this would indicate multiple arrows of different colors between two nodes; in hypertext this indicates nodes that are linked in more than one manner.
- Similarly, given a multirelational system $\mathcal{M}$ one can create a merged relational system denoted $\mathcal{R}(\mathcal{M})$ by collapsing all the different link types into one, so that

$$\mathcal{R}(\mathcal{M}) = \left\langle X, \bigcup_{i=1}^{n} R_i \right\rangle.$$

This is shown on the right of Fig. 2.

2.4 Semantic Nets

These extended multirelational structures and then typed link hypertext format is rich enough to represent semantic networks [Shastri, 1988]. These are very popular structures which are used extensively

[3] `http://www.sandia.gov/sci_compute/html_ref.html`.

in artificial intelligence and cognitive modeling. Essentially, a number of semantic categories are established, each with its own semantic properties. Concepts or nodes are then linked according to these semantic criteria. Classical semantic categories include "is-a", "has-part", "has-property", "causes", etc.,

The mapping to multirelational systems and typed link hypertext structures in general is obvious: each node of a network or each WWW page corresponds to a concept in a semantic net; and each arrow color or link type corresponds to a semantic category. In our example, we would say that v is-a c (variety is a collection), but the bibliographical reference from v to Ashby would not be represented by one of the classical semantic categories.

The different ways in which the concepts of the base sets, the ordered pairs in the relations, and the multiplicity of the relations are expressed in both multirelational systems, incidence matrices, directed graphs, and semantic networks, are shown in Tab. 1.

The closest that HTML in particular comes to supporting multirelations explicitly is the distinction between anchor links to textual nodes and links to image nodes, of the form

```
<a href="···"></a>,    <img src="···">.
```

respectively. Each of these can be considered a kind of link type. It is also possible to consider the different protocols supported by `URL`s, for example `http` for hypertext or `ftp` for file transfers, as types of links. But moreover, the kinds of navigational aids that are common in the WWW, such as "up", "down", "back", and "home", can also be considered as link types. But in this case these link types are implemented implicitly, either by a client's interface or in a particular web's architectures, and are not supported explicitly by HTML.

3 Semantic Webs

We have now developed an understanding of what semantic networks are, how they are related to hypertext corpi with multiple link types, and how they can be represented either as a set of multiple relations on a discrete space, or diagramatically as a set of nodes connected by links of different colors. We now introduce a slightly more restricted concept, that of a "semantic web".

3.1 Acyclic Multirelations and Loose Hierarchies

The key feature distinguishing semantic webs from semantic nets is that each of the binary relations present cannot have any loops or cycles.

Definition 3 (Acyclic Multirelational Systems) A relational system $\mathcal{R} = \langle X, R \rangle$ is acyclic if all paths in R are not closed. A multirelational system $\mathcal{M} = \langle X, \{R_i\} \rangle$ is acyclic if all the $\mathcal{R}_i(\mathcal{M})$ are acyclic.

$\mathcal{R}$ in Fig. 1 contains two cycles $a \rightarrow b \rightarrow a$ and $c \rightarrow c$. $\mathcal{M}$ in Fig. 2 is acyclic, but the merged graph $\mathcal{R}(\mathcal{M})$ has the cycle $a \rightarrow b \rightarrow a$.

Mathematically, this acyclicity further requires that each of the relations R_i be anti-symmetric and anti-reflexive, and thus a partially ordering on X [Preparata and Yeh, 1973]. Graphically, it requires that each set of colored links form a directed acyclic graph (DAG). Together, all of the sets of colored links can be said to form a hyper-DAG.

A DAG can be created from a simple directed graph by constructing its cyclic reduction. As an example, consider the graph $\mathcal{R}$ on the left of Fig. 3 for $X = \{a, b, \ldots, g\}$. $\mathcal{R}$ contains the cycle $b \rightarrow c \rightarrow f \rightarrow d \rightarrow b$, so let $Y = \{b, c, d, f\} \subseteq X$ be the subset of nodes containing the cycle. Then create a new relational system $\mathcal{R}' = \langle X', R' \rangle$, where:

- A new universe X' replace X, with Y replacing all the nodes in the cycle, so that $X' = \{a, e, g, Y\}$;
- The arcs strictly within Y are eliminated, and those entering or leaving Y are combined, so for example $\alpha = \langle a, b \rangle \in R$ and $\beta = \langle a, d \rangle \in R$ are combined to form $\gamma = \langle a, Y \rangle \in R'$ as shown in the center and right of the figure.

In this way a looped structure $\mathcal{R}$ is replaced by a partially ordered loopless structure $\mathcal{R}'$.

Joslyn [1991] has previously described acyclic relations (partial orderings) as "loose hierarchies": they are the most general structures which can be decomposed into distinct levels. Although they have roots and leaves, they are *not* trees, in that an element can have either multiple parents, multiple children, or both.

The levels of an acyclic graph are equivalence classes based on maximal path lengths. The level class $\Lambda(\mathcal{R}) = \{L_j\} \subseteq 2^X, 0 \leq j \leq m$ can be constructed by letting L_j be all the nodes whose maximum path length from any root is j. Thus m is the total depth of the hierarchy. So in our example

$$\Lambda(\mathcal{R}) = \{L_0, L_1, L_2, L_3\} = \{\{a\}, \{Y\}, \{e\}, \{g\}\}$$

as identified in the cyclically reduced graph on the right of Fig. 3. And note that Λ is a partition of X.

Thus the DAG structures of the diagrammatic relations reflect an intermediate form of constraint. They are certainly more constrained than a directed graph in that they are acyclic, and thus hierarchical, but only loosely so, since the nodes are only partially ordered.

3.2 Semantic Webs as Acyclic Multirelations

We define a semantic web as an acyclic semantic network. Thus no node can be mapped back to itself by the same semantic category.

There are a number of interesting consequences which follow from the acyclicity restriction, and a number of arguments for and against its usefulness. Semantically, the reflexivity of a looped structure is a kind of synonymic equivalence or identity: if a is related to b according to semantic relation R, and vice versa, then they are not distinguished by R, and are thus identical within the category.

This actually makes a great deal of sense when we consider the semantic categories used as examples

	Sets	Pairs	Mutltiplicity
Incidence matrices	Axes	Checks	Multiple matrices
Digraph	Nodes	Arrows	Colors
Hypertext	Pages	Links	Link types
Semantic network	Concepts	Semantic relation	Semantic categories

Table 1: Multiple representational forms of relational systems.

above. If a is-a b, and also b is-an a, then clearly a and b are identical; if a causes b, and also b causes a, then they are identical with respect to causation; etc.

In hypertext structures, this criterion also makes sense. Considering, for example, "forward" and "backward" as implicit link types, then if one goes forward from a to b, one cannot also go forward from b to a, but rather goes backward from b to a. Moreover, within the highly multidimensional structures which hypertext webs are, these orderings indicate single dimensions along which the web can be "cut" to allow traversal or other access in a linear form.

3.3 Indirect Cyclicity

The requirement of looplessness can be criticized, of course. In particular, the use of cyclic relations, for example feedback and autocatalytic relations, is central to Systems Science theory. And in fact, semantic webs can achieve cyclic relations, but only in a more complex, indirect way. Furthermore, this manner has much in common with the real nature of cyclicity in cybernetic systems.

Consider an acyclic multirelation $\mathcal{M}$. Acyclicity required that each of the $\mathcal{R}_i(\mathcal{M})$ is acyclic, but the merged relational system $\mathcal{R}(\mathcal{M})$ may still have cycles. The example is in Fig. 2, where each of the $\mathcal{R}_i(\mathcal{M})$ are acyclic, but still there is a cycle from a to b and back again, because $\langle a, b\rangle \in R_1$ and $\langle b, a\rangle \in R_2$. Thus the merged system $\mathcal{R}(\mathcal{M})$ on the right has a cycle $a \rightarrow b \rightarrow a$.

So in these systems loops can exist, but only when the sides of the loop cross over from one kind of relation to another. For example, in terms of semantic categories, if a is a part-of b, then b cannot also be a part-of a, rather b has-part a. Here there is a cyclic relation between a and b, but only across semantic categories, because "part of" and "has part" are inversely related categories. Or consider the example from hypertext where one goes forward from a to b, but backwards from b to a. Here a and b are indeed involved in a loop, but again only one which crosses inverse link types "forward" and "backward". More bluntly, if you can get from a to b, then you can in fact get back from b to a, but only in a *different way* from that by which you got from a to b in the first place.

This also resonates with other ideas in Systems Science about cyclicity. For example, there is a crucial asymmetry in a feedback control relation [Powers, 1989] in the form of an amplification of the energy returning on the feedback leg to the controlling action leg. And autocatalytic cycles work through the co-synthesis of two *different* compounds, each of which catalyzes the other. Thus modeling these systems requires relations of these different types. Where cyclicity exists, it is always asymmetric.

3.4 Overlapping Hierarchies

So semantic webs as acyclic multirelations have the following important properties:

Indirect Cyclicity: Paths through the web can visit themselves, but only by passing through more than one semantic relation or link type.

Overlapping Hierarchies: Each link type or semantic category when broken out by itself is a loose hierarchy, so that the web as a whole is a set of overlapping loose hierarchies.

This last point has not been discussed yet, but is best illustrated by continuing the example from Fig. 2. There $\mathcal{R}_1(\mathcal{M})$ and $\mathcal{R}_2(\mathcal{M})$ are acyclic, and thus loosely hierarchical, with level sets

$$\Lambda(\mathcal{R}_1(\mathcal{M})) = \{\{a\}, \{b, d\}, \{c\}\},$$

$$\Lambda(\mathcal{R}_2(\mathcal{M})) = \{\{b\}, \{a\}, \{c, d\}\}$$

respectively. Thus while the merged relational system $\mathcal{R}(\mathcal{M})$ on the right is cyclic, the original multirelational system $\mathcal{M}$ on the left is the union of these two loose hierarchies $\mathcal{R}_1(\mathcal{M})$ and $\mathcal{R}_2(\mathcal{M})$.

Thus in such systems multiple loose orderings or hierarchies are present simultaneously. One can traverse such a web in one general (loose) direction, for example in a chronological or alphabetical order, and then according to a different order, for example the flow of an argument or the references of a particular author.

This is a central idea for the organization of the Principia Cybernetica web, where these ordering are called "guides". The linear order provided by the table of contents, for example, is just one such possible ordering among many. Others can be generated by each of the editors of the project, by collaborators, etc.

Continuing now in Fig. 4, one can construct the cyclic reduction $\mathcal{R}'(\mathcal{M})$ of the merged relational system $\mathcal{R}(\mathcal{M})$, as shown on the left, where now $Y = \{a, b\}$. This is clearly a DAG, with levels as shown. And when the reduction of X to $X' = \{Y, c, d\}$ is reflected into the two constituent loose hierarchies, the resulting systems $\mathcal{R}_1(\mathcal{M})$ and $\mathcal{R}_2(\mathcal{M})$ are still loose hierarchies, as shown in the center. Finally, the cyclic reduction $\mathcal{M}' = \mathcal{R}_1'(\mathcal{M}) \cup \mathcal{R}_2'(\mathcal{M})$ through $\mathcal{R}(\mathcal{M})$ of the original system $\mathcal{M}$ is shown on the right.

4 The Significance for Cyberspace and Systems Science

The significance of semantic networks and webs for both cyberspace and Systems Science cannot be underestimated. The fact is, both cyberspace and Systems Science are inherently suitable and appropriate for each other. As Systems Science is the ultimate transdisciplinary field, so its works have always tended towards the encyclopedic. Serious development of the kinds of synthetic, syncretic, broadly unifying theories in which Systems Scientists indulge is dependent exactly on the kinds of representational forms that cyberspace, and particularly semantic webs and networks, provides.

The connections between ideas in, say, sociology and chemistry are complex and intricate. Furthermore, each researcher brings their own interpretations, and there are many side-paths and connections to other areas along the way. Each of these kinds of relations is naturally represented as another link type, and, as argued above, within each link type loopless structures are preferred.

Historically, of course there are many precedents for the use of semantic networks and webs in Systems Science. Certainly the many efforts to create glossary or dictionary-type works for Systems Science must be included, for example the work of François [1992]. But beyond that, there is a rich tradition in Systems Science work of the use of structures very similar to semantic webs, for example in Heylighen's distinction dynamics [Heylighen, 1991] or Pask's entailment meshes [Pask, 1980] (an example of their use is in the crucial *Cybernetics of Cybernetics* anthology [von Foerster, 1979]). Rosen has made extensive use of structure quite similar to semantic webs [Rosen, 1991], building for a category theoretical perspective. In particular, his relational definition of the organism can be cast as a semantic web. And of course, the participants in the PRINCIPIA CYBERNETICA Project are trying to explicitly implement and develop these ideas and other very similar ones.

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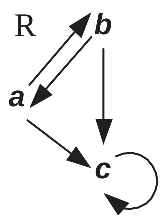

Fig. 1: The directed graph of a relational system.

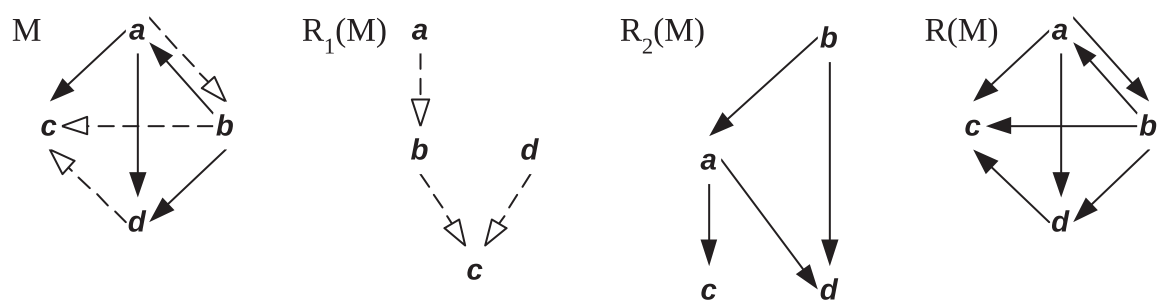

Fig. 2: (left) A multirelation. (center) Its two constituent relations.
(right) The merged relation.

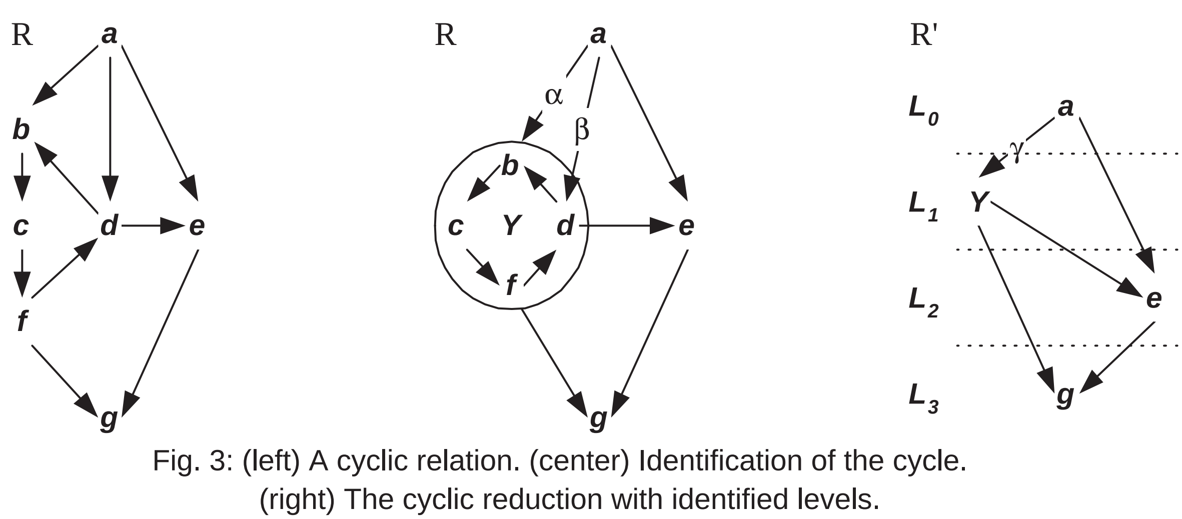

Fig. 3: (left) A cyclic relation. (center) Identification of the cycle.
(right) The cyclic reduction with identified levels.

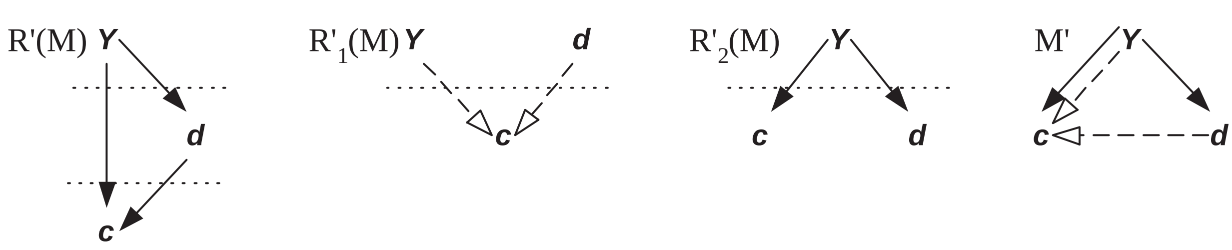

Fig. 4: (left) The cyclic reduction of R(M). (center) Its two constituent relations.
(right) The acyclic multirelation.