Report from Dagstuhl Seminar 11371

Uncertainty modeling and analysis with intervals: Foundations, tools, applications

Edited by

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Abstract

This report documents the program and the results of Dagstuhl Seminar 11371 "Uncertainty modeling and analysis with intervals – Foundations, tools, applications", taking place September 11-16, 2011. The major emphasis of the seminar lies on modeling and analyzing uncertainties and propagating them through application systems by using, for example, interval arithmetic.

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1 Executive Summary

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Verification and validation (V&V) assessment of process modeling and simulation is increasing in importance in various areas of application. They include complex mechatronic and biomechanical tasks with especially strict requirements on numerical accuracy and performance. However, engineers lack precise knowledge regarding the process and its input data. This lack of knowledge and the inherent inexactness in measurement make such general V&V cycle tasks as design of a formal model and definition of relevant parameters and their ranges difficult to complete.

To assess how reliable a system is, V&V analysts have to deal with uncertainty. There are two types of uncertainty: aleatory and epistemic.

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are NP-hard and only a formula using exponential number of ordinary linear programs is known. The second fundamental problem, which is very difficult and still challenging, is to find a tight enclosure of the optimal solution set. This becomes easy in the case of the so called basis stability, i.e., there is a basis optimal for each realization of interval data. Checking basis stability is also a computationally hard problem, but there are quite strong sufficient conditions that may be utilized. Eventually, we state some open problems.

References

1 Milan Hladík. *Linear Programming – New Frontiers in Theory and Applications*, 2nd chapter Interval linear programming: A survey. Nova Science Publisher, New York, 2011.

3.7 Intervals, Orders, and Rank

Cliff Joslyn (Pacific Northwest National Lab. – Seattle, US)

Joint work of Joslyn, Cliff; Hogan, Emilie

Intervals are ordered
Orders have intervals
Orders have ranks
Ranks are intervals

Intervals have order relations defined on them as an important operation. Also, partially ordered sets (posets) and lattices have intervals within them as important sub-structures. In traditional interval analysis, the set on which the intervals are drawn is the real numbers, a special ordered set which is a total order; and the ordering relation used between these real intervals is the "strong" interval order (one interval being entirely below another), in the context of an overall Allen's algebra.

But in our work in semantic databases and ontology management, it is the more general cases which are demanded. Specifically: 1) the intervals in question are valued in a finite, bounded, generally partially ordered set; and 2) when real intervals ARE used, the conjugate endpoint product order and subset orders are far preferable to the standard strong order (which isn't even really an ordering relation anyway). The attendant issues have implications for the foundations of interval analysis which we seek to explore with the group.

Depending on time, structure, and the interests of attendees, we can go into more or less depth on the following.

We begin by describing our use of large, finite, bounded posets to represent taxonomic semantic data structures for applications such as ontology clustering and alignment. We then consider the challenges presented by their layout and display.

Our first challenge, the vertical layout of nodes, we have been working on for a while. We observe that rank in posets is best considered as being valued on integer intervals. These integer-valued rank intervals can themselves in turn be ordered (in the endpoint product order), so that an iterative operation is available. Repeated application serves to identify a privileged embedding of the poset to a total preorder preferred to reflect the underlying partial order. We have results about how the height, width, and dimension of the poset changes in repeated application, and prove that we do achieve a final embedding of the original poset to a total preorder. In the process, results about measures of gradedness of posets are also motivated.

Our second challenge we are just beginning to explore, but we will present some preliminary ideas for discussion. We seek to simplify the display of large posets (actually lattices in the first treatment) when only a subset of nodes are specified by a user. A tremendous reduction in complexity can be available when (poset) intervals among the target nodes are identified which are disjoint (pairwise or moreso). The underlying mathematical representation suggested is the graph of the intersection structure of poset intervals, that is, a generalization of an interval graph to poset intervals. Cliques in this graph determine the number of total meets and joins which need to be displayed in the reduced visualization, and thus the amount of compression achievable. But, this approach requires us to have the ordering relation between pairs of poset intervals, that is, to develop an Allen's algebra generalized to poset intervals.

3.8 Interval Computations – Introduction and Significant Applications

Ralph Baker Kearfott (Univ. of Louisiana – Lafayette, US)

In this tutorial, we first outline the historical motivations and early work in interval arithmetic, then review basic interval arithmetic operations and their properties, including advantages and pitfalls. We conclude with a variety of examples of successful application of interval arithmetic.

Historical motivation and work includes

1931 — Classical analysis: Rosaline Cecily Young developed interval arithmetic to handle analysis of one-sided limits (where $\liminf f \neq \limsup f$) [15].

1951 — Roundoff error analysis: Paul S. Dwyer developed interval arithmetic in the chapter on roundoff error analysis in his numerical analysis text [3].

1956 — Calculus of Approximations: Warmus and Steinhaus developed interval arithmetic to provide a sound theoretical backing to numerical computation [14].

1958 — Automatic error analysis: Teruro Sunaga developed interval arithmetic [11].

1959 — Automatic error analysis: Ray Moore developed interval arithmetic in a report and dissertation to which most modern work on the subject can be traced [5].

Advantages of interval arithmetic include the ability to quickly compute mathematically rigorous bounds on roundoff error and on ranges of functions, where computation of the exact range is NP-hard. Disadvantages are that these bounds may be unusably pessimistic, unless special algorithms are designed.

Current successful significant applications include the following:

A filter in branch and bound methods in leading commercial software, such as [9] (and others).

Constraint solving and constraint propagation, as in [8, §14] and numerous other works.

Verified solution of ODEs, as in [1], [4], etc.

Computer-aided proofs, as in [13], [12].

Chemical engineering, as in [10].

PDE problems such as structural analysis with uncertainties [6], [7] or analysis of photonic crystals [2].

Numerous others.