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HIERARCHY AND STRICT HIERARCHY IN GENERAL INFORMATION THEORY*

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Abstract

We consider the concepts of *strict* and *loose* hierarchy, where a loose hierarchy has some degree of overlap among sub-systems at a level. We describe the properties of loose hierarchies and loose structural hierarchies, which are based on set inclusion and intersection; and we suggest a measure of the looseness of a random set, as used in Generalized Information Theory.

Keywords: Hierarchy, loose hierarchy, levels, evidence theory, generalized information theory, random sets, fuzzy sets.

An Informal Description of Strict and Loose Hierarchy

The traditional concept of a "hierarchy" is of an overall system composed of sub-systems, and those in turn of sub-sub-systems, etc. Bunge [6] reminds us that the etymology of "hierarchy" is based on the levels of ecclesiastical authority, and rejects the usage of the term in its now traditional form. Instead, he prefers the concept of a "level structure" as a partially ordered organization. Similarly, Webster defines the modern sense of "hierarchy" as "a graded or ranked series" [12] or "persons or things arranged in a graded series" [13].

We will adopt the spirit of this approach and describe a hierarchy as: "a structure which admits of description in terms of levels".[1] This is turn leads us to accept Steven Rogers' definition as perhaps the best available in the context of Systems Science:

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[1] By using the phrase "description in terms of levels" we are offering an epistemic definition, but this is not meant to imply that an ontological claim *cannot* be made: systems which are in some sense "really" hierarchical should be able to be so described [17, 19]. This argument is not the subject of this paper.

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> A partially-ordered structure of entitites in which every entity but one is successor to at least one other entity; and every entity except the basic entities is a predecessor to at least one other entity. [3]

It must be emphasized, however, that this traditional concept does *not* entail another set of properties that most hierarchy researchers generally assume: that the elements at one level form a *partition* of that level; that the overall structure is a graph-theoretic *tree*; and that each sub-system participates in exactly one branch of that tree.

A brief review of the literature on hierarchy theory reveals that this idea is quite common, and frequently left unspecified [2, 5, 14, 21]. For example, von Bertalanffy writes:

> A general theory of hierarchic order obviously will be a mainstay of general systems theory. In graph theory hierarchic order is expressed by the "tree", and relational aspects of hierarchy can be represented in this way. [25, p. 28]

Simon offers the criteria of "near decomposability" as necessary for hierarchy:

> ... we may move to a theory of *nearly decomposable* systems, in which the interactions among the subsystems [vs. the interactions *within* the subsystems] are weak, but not negligible. [22, p. 69]

This condition leads to a "loose coupling" of sub-systems from each other both "vertically" (between levels) *and* "horizontally" (at one level) from each other. This combination of vertical and horizontal separation leads Simon to consider hierarchy as requiring the partitioning of each level, and thus the classic tree structured hierarchy:

> While the ordinary sequence of Chinese boxes is a sequence, or complete ordering, of the component boxes, a hierarchy is a partial ordering — specifically, a tree. [23, p. 5]

The criteria of near decomposability is echoed by Auger in the context of hierarchies resulting from physical aggregation:

> We assume that the inter-group distances are always very large with respect to intra-group distances between elements. [4, p. 1]

He uses similar criteria for other hierarchies, and is led to conclusions that are similar to Simon's.

There is no doubt that trees can be described in terms of levels, and are thus hierarchies. But there are many other partially ordered structures which meet Rogers' definition, and can also be described in terms of levels. We will call these cases "loose hierarchies", as distinguished from "strict hierarchies", which are trees, and limiting cases of the general hierarchies.[2]

[2] We will *not* use the term "heterarchy" for "loose hierarchy" because it has been used in a variety of ways, and lacks a cohesive sense [11, 21], although Minsky's use [16] is close to ours.

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A loose hierarchy is distinguished by sufficient interaction among sub-systems *within a level* such that there are some elements that cannot be clearly identified as belonging to one sub-system or another, and so that there is *overlap* among these sub-systems. In a formal hierarchy, we would say that there is a non-empty intersection between sub-systems at one level; in a structural hierarchy, that elements belong to multiple sub-systems; in a functional hierarchy, that elements serve multiple functions; in a process hierarchy, that there are simultaneous activities; and in a taxonomic hierarchy, that there are fuzzy boundaries between taxonomic groups. There are many who have held this view of general hierarchical structures, and for them, hierarchies are partially ordered structures which are much more general than trees [1, 6, 9, 18, 20, 15, p. 644].

We note a number of things about loose hierarchies:

- We fully agree that hierarchy requires a clear separation of *levels* from each other (*vertical* near decomposability), but hold that it does *not* require a clear separation of *sub-systems* from each other at *one* level (*horizontal* near decomposability).
- We are considering overlap between sub-systems at one level, and not sub-systems at different levels, nor interaction between "levels" themselves.
- Not just *any* interactions among sub-systems at one level is sufficient for loose hierarchy. The interaction must be sufficent to bring into question the actual *boundaries* between the sub-systems, and to consider a different description of the *structure*, such that the sub-systems overlap. For example, this rules out the kinds of intra-level interactions considered by Simon [22] and Auger [4], since they are not sufficient to destroy horizontal near decomposability. Loose hierarchies are *not at all* decomposable in the horizontal dimension.
- Finally, we note that the overlap, and thus the "looseness" of a hierarchy, can come in *degrees*, and may be *quantifiable* under a given system description. Thus hierarchy itself is a concept which admits degrees. A system can be *more or less* hierarchical: perhaps *completely* hierarchical (strict hierarchy), *somewhat* hierarchical (loose hieararchy), or *not at all* hierarchical (no hierarchy).

In the following sections, we will first consider the formal properties of hierarchies. Then we will describe the properties of structural hierarchies, which are based on set inclusion and intersection. Finally, we will consider a random set as a probability measure on a structural hierarchy, and suggest a measure of the looseness of a random set's hierarchical structure.

A Formal Description of Strict and Loose Hierarchy

Consider a countable universe of discourse $U = \{x_i\}$ and relation $R \subset U^2$ with transitive closure R^T. We will regard the $x_i \in U$ as nodes, and a pair $x_1 R x_2$ as an arc from node x_1 to node x_2. We then define a general hierarchy as:

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Definition 1 (Hierarchy) *R is a hierarchy iff R is connected and R^T is anti-symmetric.*

The connectedness criteria requires that all elements of U which participate in the hierarchy participate in the *same* hierarchy. We note that R^T is a non-reflexive partial order, denoted as $>$, that is $x_1 > x_2$ iff $x_1 R^T x_2$. Since $R \subset R^T$, R itself must be anti-symmetric, and so the definition guarantees that there are no cycles of any length in R. All paths through R are linear, and so R can be represented as a directed acyclic graph (DAG). This is the essential property of hierarchies: levels requires a lack of loops, and vice versa.

It is common to require that a hierarchy have a unique greatest bound, called the *root* node. We have the following definiton, which we believe is formally equivalent to Rogers' above:

Definition 2 (Rooted Hierarchy, Rogers) *R is a rooted hierarchy with root $x_0 \in U$ iff R is a hierarchy and $\forall x_i \neq x_0, x_0 > x_i$.*

The further condition that is necessary to derive R as a *strict* hierarchy is that *each non-root node have exactly one parent*, and thereby participates in a unique path from the root:

Definition 3 (Strict Hierarchy) *R is a strict hierarchy iff R is a rooted hierarchy and $\forall x_i \neq x_0, \exists! x' \in U, x' R x_i$.*

Finally, a loose hierarchy is a non-strict hierarchy:

Definition 4 (Loose Hierarchy) *R is a loose hierarchy iff R is a rooted hierarchy and R is not strict.*

Thus, in a loose hierarchy there are some nodes which have multiple parents.

The levels of a hierarchy R are a linearly ordered partition of R based on the path lengths. Fig. 1 illustrates the concepts of both strict and loose rooted hierarchies and their levels.

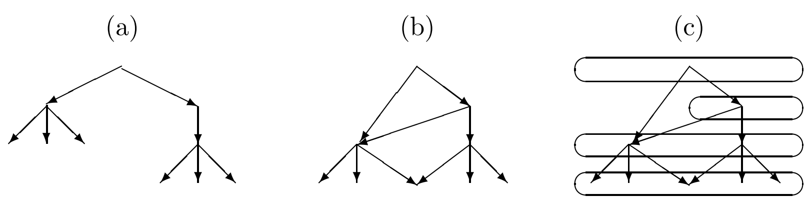

Figure 1: (a) A strict hierarchy; (b) A similar loose hierarchy, where some nodes have multiple parents; (c) The resulting level structure.

An Example: Type Inheritance

An example of strict vs. loose hierarchy is provided by the data-typing systems of "object-oriented" computer languages. In such languages, a "class" is an intelligent, abstract data type. A class D can be *derived from* a *base class* B, in which case it *inherits* the attributes of class B. In a *single inheritance* language, a class D can only be derived from a single base B. In a *multiple inheritance* language, D can be derived from multiple base classes $B_1, B_2, \ldots, B_n$, thereby inhereting attributes from all the B_i.

Single inheritance is a strictly hierarchical data-typing system; multiple inheritance is loosely hierarchical. Multiple inheritance has proved critical for a language to provide a general environment for system representation. For example, C++ [24] was originally introduced as an object-oriented programming language with single inheritance, but multiple inheritance was quickly added as a critical attribute of the language.

Loose Structural Hierarchies

The application of the above principles to a typical part-whole hierarchy is straightforward. We will define a *structural hierarchy* as a rooted hierarchy in which sub-systems (parts, entities at one level) are completely contained in super-systems (wholes, entities at levels above them). So, in a *strict* structural hierarchy, each part (except the root, the "greatest whole") will be contained in exactly one whole at an immediately higher level. If we interpret this "containment" as set inclusion, then we must move to interpret the nodes in R as *subsets* of U, and the level ordering is provided by the partial ordering of the $\subset$ relation on subsets of U.

Formally, let $\mathcal{C} = \{F\}, \forall F, F \neq \emptyset, F \subset U$ be a class of non-empty subsets of U. We must ensure that $\mathcal{C}$ contains the root U, so let $\mathcal{C}^* = \mathcal{C} \cup \{U\}$. Then $\mathcal{C}^*$ is a structural hierarchy, and for any two subsets $F_1, F_2 \in \mathcal{C}^*$, there are three possible situations, denoted by three symmetric operators:

Inclusion: $F_1 \bowtie F_2 =_{def} F_1 \subset F_2$ or $F_2 \subset F_1$.

Disjoint: $F_1 \perp F_2 =_{def} F_1 \cap F_2 = \emptyset$.

Overlapping (Properly Intersecting): $F_1 \circ F_2 =_{def} F_1 \not\perp F_2$ and $F_1 \not\bowtie F_2$.

When $F_1 \subset F_2$ (resp. $F_2 \subset F_1$), then they are related by *depth*, and F_1 is on a path at a lower (higher) level than F_2. When $F_1 \perp F_2$, then they are related by *breadth*, and there is no path between F_1 and F_2. While they may be parts of some common higher system, neither is a part of the other. Finally, when $F_1 \circ F_2$, then the set $F_1 \cap F_2$ is the proper overlap of F_1 and F_2, and is properly a part of both F_1 and F_2.

Now construct $\mathcal{C}'$ by extending $\mathcal{C}^*$ to include all the non-empty intersections of the $F_i, F_j \in \mathcal{C}^*$ taken pairwise:

$$\mathcal{C}' = \mathcal{C}^* \cup \{F_1 \cap F_2 : F_1, F_2 \in \mathcal{C}^*, F_1 \not\perp F_2\}. \tag{1}$$

$\mathcal{C}'$ is a structural hierarchy on the relation $\subset$, and clearly $\mathcal{C}'$ is a strict structural hierarchy iff $\mathcal{C}' = \mathcal{C}^*$, that is iff $\forall F_1, F_2 \in \mathcal{C}^*, F_1 \not\circ F_2$. So $\mathcal{C}'$ is a loose structural hierarchy iff $\exists F_1, F_2 \in \mathcal{C}^*, F_1 \circ F_2$. We will say that $\mathcal{C}^*$ is a strict (loose) structural hierarchy whenever its extension $\mathcal{C}'$ is.

Thus, in general we can construe two distinct ways in which sets can be related in a strictly hierarchical manner: they can be either inclusive or disjoint. In all other situations, a loosely hierarchical class results. For example (fig. 2), let $|U| = 4$ and $\mathcal{C}^* = \{\{x_1, x_2, x_3, x_4\}, \{x_1, x_2\}, \{x_2, x_3\}, \{x_1\}, \{x_4\}\}$. To construct $\mathcal{C}'$, $\{x_1, x_2\} \cap \{x_2, x_3\} = \{x_2\} \notin \mathcal{C}^*$ must be added to $\mathcal{C}^*$, and therefore $\mathcal{C}^*$ is a loose structural hierarchy.

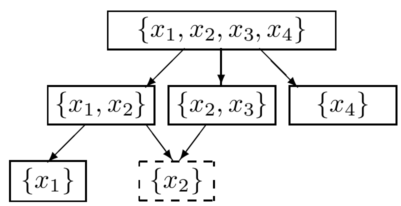

Figure 2: An example of a loose structural hierarchy, where $\{x_2\} \in \mathcal{C}', \{x_2\} \notin \mathcal{C}^*$.

Strict Hierarchy in Generalized Information Theory

Our immediate interest and impetus for developing this work is the use of these concepts in the Dempster-Shafer theory of evidence. This theory forms one part of the core of Generalized Information Theory, along with Fuzzy Sets and Possibility Theory [10].

An *evidence measure* is a function $m\colon 2^{(}U) \mapsto [0,1]$ on the subsets of U with $m(\emptyset) = 0$ and $\sum_{F \subset U} m(F) = 1$. A *random set* is a set of tuples $\mathcal{S} = \{\langle F_j, m_j \rangle : m_j > 0\}$, where $F_j \subset U, m_j = m(F_j)$, and $1 \leq j \leq |\mathcal{S}| \leq 2^{|U|} - 1$. We also have the *focal set* $\mathcal{F} = \{F_j : m_j > 0\}$.

Given a random set $\mathcal{S}$, then there are two dual belief and plausibility measures defined on each $F \subset U$:

$$\mathrm{Bel}(F) = \sum_{F_j \subset F} m_j = 1 - \mathrm{Pl}(\bar{F}) \tag{2}$$

$$\mathrm{Pl}(F) = \sum_{F_j \not\perp F} m_j = 1 - \mathrm{Bel}(\bar{F}). \tag{3}$$

Since $\mathcal{F}$ is a class of U, $\mathcal{F}^*$ is a structural hierarchy, and we can apply the ideas developed above. In particular, we note that there are two special cases of strict structural hierarchy: extremes of *depth* and *breadth*.

Probability: When $\forall F_1, F_2 \in \mathcal{F}, F_1 \perp F_2$ and $\bigcup_{\mathcal{F}} F_j = U$, then $\mathcal{F}$ *partitions* U. $\mathcal{F}^*$ is a strict structural hierarchy which is completely broad and minimally deep: $|\mathcal{F}|$ branches of depth 2.

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The most refined partition of U is when $\mathcal{F}$ is *specific*, so that $\forall F_j \in \mathcal{F}, \exists x_i \in U, F_j = \{x_i\}$. Now $\mathcal{F}^*$ is a strict structural hierarchy with maximal breadth and minimal depth: $|U|$ branches of depth 2. Under these conditions, $\mathrm{Bel}(F_j) = \mathrm{Pl}(F_j) = \mathrm{Pr}(F_j)$ is a *probability measure* on $\mathcal{F}$.

Possibility: When $\forall F_1, F_2 \in \mathcal{F}, F_1 \bowtie F_2$, then $\mathcal{F}$ is a *nest* of U. $\mathcal{F}^*$ is a strict structural hierarchy which is completely deep and minimally broad: one branch of depth $|\mathcal{F}|$. Also, $\mathrm{Pl}(F_j) = \Pi(F_j)$ is a *possibility measure* on $\mathcal{F}$. If $\mathcal{F}$ is a *complete* nest (assuming, without loss of generality, an ordering of the $x_i \in U$, then $\forall x_i, \exists F_j = \{x_1, x_2, \ldots, x_i\} \in \mathcal{F}$), then $\mathcal{F}^* = \mathcal{F}$ is a strict structural hierarchy with maximal depth and minimal breadth: one branch of depth $|U|$.

These are two extreme cases, but there are also mixed cases where $\mathcal{F}^*$ is a strict structural hierarchy. In these cases $\mathcal{F}$ contains some pairs of subsets which are nested, others which are disjoint, but none which have a proper overlap $F_1 \circ F_2$.

We will not discuss Generalized Information Theory or the relationship between probability and possibility further in this paper, but rather refer the reader to the extensive literature on the subject [7, 8, 10]. Suffice it to say that a possibilistic information theory is under development, complete with information measures of "nonspecificity" which play the same role as entropy in stochastic information theory.

A Measure of Loose Hierarchy in Generalized Information Theory

Our goal is to develop a formal measure of the looseness Ł of the hierarchy of a random set $\mathcal{S}$. $\text{Ł}(\mathcal{S})$ should be zero (no looseness) when $\mathcal{F}^*$ is a strict hierarchy (e.g. $\mathcal{S}$ is a probabilistic or possibilistic random set), and should increase from zero as the "quantity" of overlap (or "looseness") increases.

We can move towards $\text{Ł}(\mathcal{S})$ by first developing a pairwise measure $\text{Ł}(F_1, F_2)$ for subsets $F_1, F_2 \in \mathcal{F}$. The measures $\text{Ł}(F_1, F_2)$ and $\text{Ł}(\mathcal{S})$ are still under development, but we can say that $\text{Ł}(S)$ will be an aggregation of the $\text{Ł}(F_1, F_2)$. Some desirable properties of $\text{Ł}(F_1, F_2)$ can be dervied from the following table, which assumes non-empty F_1, F_2:

	$\|F_1 - F_2\|$	$\|F_2 - F_1\|$	$\|F_1 \cap F_2\|$
$F_1 \subset F_2$	0	≥ 0	$\|F_1\| > 0$
$F_1 \supset F_2$	≥ 0	0	$\|F_2\| > 0$
$F_1 \perp F_2$	$\|F_1\| > 0$	$\|F_2\| > 0$	0
$F_1 \circ F_2$	> 0	> 0	> 0
$F_1 = F_2$	0	0	$\|F_1\| = \|F_2\| > 0$

When for each pair $\langle F_1, F_2 \rangle$, the class $\{F_1, F_2\}^*$ is a strict structural hierarchy, then the cardinalities of one of $F_1 - F_2, F_2 - F_1$, or $F_1 \cap F_2$ must be zero, and we require $\text{Ł}(F_1, F_2)$ to be zero. Also, $\text{Ł}(F_1, F_2)$ should reach its maximum when F_1 and F_2 are maximally overlapping, that is $|F_1 - F_2| = |F_2 - F_1| = |F_1 \cap F_2| > 0$.

For illustration (fig. 3), let $|F_1| < |F_2|$, and consider first $F_1 \perp F_2$, so that $|F_1 \cap F_2| = 0$. Then F_1 and F_2 are at the same level in $\{F_1, F_2\}^*$, the hierarchy is strict, and $\text{Ł}(F_1, F_2) = 0$. As we move to $F_1 \circ F_2$, then $0 < |F_1 \cap F_2| < |F_1|$, the hierarchy is loose, and $\text{Ł}(F_1, F_2) > 0$. Finally, when $F_1 \subset F_2$, then $|F_1 \cap F_2| = |F_1|$, F_1 is at a lower level than F_2, the hierarchy is once again strict, and $\text{Ł}(F_1, F_2) = 0$.

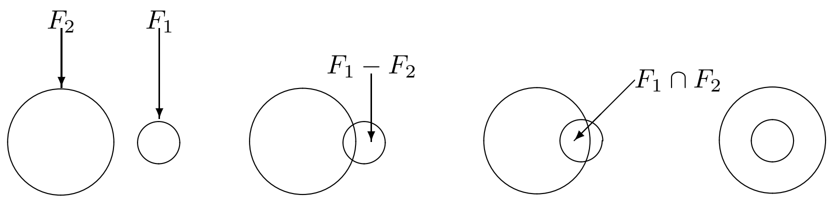

Figure 3: Movement of F_1 into F_2 from disjointness through overlap to inclusion: $|F_1 \cap F_2|$ grows from 0 to $|F_1|$ while $|F_1 - F_2|$ shrinks from $|F_1|$ to 0.

The other desirable properties of $\text{Ł}(F_1, F_2)$ depend on the relative cardinalities $|F_1 - F_2|, |F_2 - F_1|$, and $|F_1 \cap F_2|$. Is the pair F_1, F_2 "looser" when $|F_1 \cap F_2|$ is large relative to $|F_1 - F_2|$ or $|F_2 - F_1|$, or vice versa (fig. 4) If we weight them equally, then there are still a number of different possible

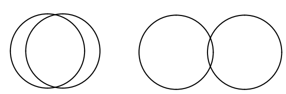

Figure 4: Two other possibilities for $|F_1| = |F_2|$, but $|F_1 \cap F_2|$ either small or large.

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functions $\text{Ł}(F_1, F_2)$, for example:

$$\text{Ł}(F_1, F_2) = |F_1 - F_2| \cdot |F_2 - F_1| \cdot |F_1 \cap F_2| \tag{4}$$

$$\text{Ł}(F_1, F_2) = \min\left(|F_1 - F_2|, |F_2 - F_1|, |F_1 \cap F_2|\right) \tag{5}$$

Given a function $\text{Ł}(\cdot, \cdot)$, then $\text{Ł}(\mathcal{S})$ can be developed by aggregating, for all F_j, the m_j weighted by the quantity of overlap with all other F_k and normalized by $|F_j|$:

$$\text{Ł}(\mathcal{S}) = \sum_j \frac{m_j}{|F_j|} \sum_{F_k \in \mathcal{C}} \text{Ł}(F_j, F_k) \tag{6}$$

Conclusion

We have considered the concept of the "looseness" of a hierarchy, both formally and informally, and its application to the structural hierarchy of random sets. The measure $\text{Ł}(\mathcal{S})$ requires further work, including a more thorough justification and comparison with other methods based on other pairwise functions $\L(F_1, F_2)$.

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