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# ON THE SEMANTICS OF ENTROPY MEASURES OF EMERGENT PHENOMENA

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This paper is an attempt to perform some semantic analysis of the concept of "entropy" as it is used in both thermodynamic and nonthermodynamic contexts. We first consider the relation between thermodynamic entropy as a semantic concept, a measured quantity of real systems; and statistical entropy as a syntactic concept, a calculated quantity in formal systems. We then consider the use of entropy as a measure of different types of systems, the origin and identification of emergent levels in physical systems, and the ontological status of nonthermodynamic entropies.

## SYNTACTIC vs. SEMANTIC DEFINITIONS OF ENTROPY

The literature on the relation between thermodynamics and information theory is vast (Brillouin 1964, Carnap 1977, Haken 1988, Jaynes 1957, Shannon and Weaver 1964, Wicken 1987). While this debate is not the central concern of this paper, in any discussion of entropy it is wise to clearly express how it will be understood. We do so on the basis of the relation and difference between quantities that result from *measurements* of real systems and quantities that result from *calculations* in formal, mathematical systems. Science is (at least) the interaction between theory and observation. When theory is seen as a model of reality, its function is to predict (retrodict) future (past) observations. In the present context, we are dealing with two different models of parts of reality. The thermodynamic definition of entropy

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$$dS = \frac{dQ_{\mathrm{rev}}}{T} \tag{1}$$

serves to measure the thermodynamic state of a thermodynamic system, where $Q$ is heat and $T$ is temperature. On the other hand, the statistical mechanical definition of entropy is the Boltzman entropy:

$$H_{\mathrm{Boltz}}(\vec{p}) = -k_{\mathrm{b}} \langle \log_{10}(p_i) \rangle \tag{2}$$

where the vector $\vec{p} = (p_i)$, $1 \leq i \leq n$, is a probability distribution with $p_i$ being the probability of the mechanical (quantum-mechanical) system having a certain energy level; $k_{\mathrm{b}} = 1.38 \times 10^{-22}$ J/T is the Boltzman constant; and $\langle \cdot \rangle$ denotes a mean over the distribution $\vec{p}$. $H_{\mathrm{Boltz}}$ serves as a model of the mechanical system that predicts values for the measured quantity $S$ in equation 1 under equilibrium conditions (Rosser 1982). $H_{\mathrm{Boltz}}$ is a specific form of a general statistical entropy:

$$H(\vec{p}) = -k \langle \log_a(p_i) \rangle \tag{3}$$

where $\vec{p}$ is any probability distribution and $k$ and $a$ are free parameters. Thus, the selection of $k$ and $a$ establishes a scaling on $H$. $k = 1$ and $a = 2$ is another common scaling, used in information theory to give the Shannon entropy:

$$H_{\mathrm{Shan}}(\vec{p}) = -\langle \log_2(p_i) \rangle \tag{4}$$

Now the ontological status of the two quantities $S$ and $H$ is very different. Thermodynamic entropy is a property of certain kinds of real systems: thermodynamic systems. Thermodynamic entropy is a measured property of such systems, and is understood in a differential relation to other quantities, such as heat, work, and temperature, which are also measured on thermodynamic systems.

Thermodynamic entropy is thus a "content-full" concept specific to thermodynamic systems. The semantics of thermodynamic entropy is necessarily deeply embedded within the body of thermal physics, and it must be interpreted in the context of all of thermodynamics. In particular, it must be interpreted within the results of thermodynamics and other thermodynamic quantities such as energy, pressure, and temperature. In this context, the laws of thermodynamics, in particular, the second law, are powerful and significant results.

But unlike thermodynamic entropy, statistical entropy is a property of a probability distribution, not a real system. Statistical entropy is calculated from the numerical properties of that distribution. It is understood as the establishment of a many-to-one relation between two sets, so that the entropy is not distinguished among members of equivalence classes of the domain. So the *measurement* of an entropy quantity is the converse relation, a one-to-many relation which identifies that equivalence class of states from the domain consistent with the measured entropy. In the case of the $H_{\text{Boltz}}$ statistical mechanical entropy, the many-to-one relation is established from the set of mechanical states to the set of thermodynamic states; and in the general statistical entropy $H$ from the set of probability distributions to the nonnegative real numbers.

Statistical entropy is thus essentially a "content-free" concept. Whatever interpretation of the $p_i$ we make need only adhere to the axioms of probability theory. The measure itself is chosen based on the well-known axioms of its definition (Aczel and Daroczy 1975, Klir and Folger 1987, p. 156).[1]

Unlike the definition of thermodynamic entropy, in the definition of statistical entropy the basic quantities in question (the $p_i$) are not interpreted with respect to a specific semantic domain. Thus, statistical entropy lacks any inherent semantics and, in isolation, is a purely syntactic concept. Furthermore, since statistical entropy, in particular, does not involve the context of thermal physics, the simple usage of an entropy function does not entail any second law of thermodynamics or any "second law correlate."

We can construct a model of some part of nature when we choose to

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[1] It should be noted that there is current interest in the use of alternative formalisms where some of those axioms are relaxed. For example, relaxing the additivity requirement in the definition of statistical entropy results in the class of Renyi entropies of the form

$$H_\alpha(\vec{p}) = \frac{1}{1-\alpha} \log_2 \sum_i p_i^\alpha$$

for $\alpha \neq 1$ ($\lim_{\alpha \to 1} H_\alpha = H_{\text{Shan}}$). This formula has been applied with some success to measure complexity (Lindgren and Nordahl 1988) and in dynamical systems in relation to the "subjectivity" of an observer (Jumarie 1990). Also, relaxation of the additivity requirement in the basic probability formalism results in whole new classes of uncertainty measures (Klir and Folger 1987, Shafer 1976). And similar axioms used in the context of *possibility* distributions lead to an alternative information theory (Dubois and Prade 1988).

interpret the $p_i$ in some specific context and assign values to $k$ and $a$. So in statistical mechanics we regard the $p_i$ as probabilities for a (quantum) mechanical system having a certain energy level. Under these conditions, $H_{\mathrm{Boltz}}$ calculates the measured value of $S$ in equation 1 *because statistical mechanics is a good theory for thermodynamics.* But there is nothing necessary about this relation; indeed, the scientific significance of the theory arises just because it might *not* have been the case.

In another situation, however, we can interpret the $p_i$ in some other way, for example, the chance that a certain symbol will be received at the end of a communications channel (communications entropy); the "density" of distribution of orbit traversal of a dynamical phase space (metric entropy, topological entropy) (Schuster 1984); the chances for mating within a certain population (cladistic entropy) (Brooks and Wiley 1988); or the variety in the distribution of cell wall lengths and intersection angles in cellular structures like plant tissues and soap foams (River 1986, Weaire and Rivier 1984).

The purpose of the work pioneered by Shannon and Weaver (1964), Jaynes (1957) and Rosenkrantz (1989) is to apply the entropy measure in as many different contexts as possible.[2] Now in these cases, equation 2 can generally no longer be used to predict thermodynamic measurements, and a second law correlate does not necessarily exist. Thus, we can understand the effect of the Shannon–Jaynes program in information theory as a generalization of the concept of entropy away from a *semantic* basis of a content-full specific interpretation to a *syntactic* basis of a content-free formalism. For them, the meaning of the entropy quantity is not significant, only its form. They use and understand entropy without regard to the domain of application or the real system, if any, on which the quantity might be measured. They have a *weaker* criteria for establishing an entropy measure: Only the axioms of the formalism, and not the results of some domain of interpretation, need apply.

This distinction is of great importance, but is easy to overlook. It is not my interest to claim one usage or the other as being either "correct" or "superior." Perhaps such decisions must rest on convenience, taste, or historical prejudice. But the second law of thermodynamics is not the second law of entropy, and ignorance of these different criteria can lead to serious error and confusion.

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[2] This work is being continued and generalized (Christensen 1980, Jumarie 1990, Kapur 1989, Skilling 1989).

## ENTROPY AS A TYPE-SPECIFIC MEASURE

A primary goal for interdisciplinary scientists and systems scientists is finding good theories that hold for different kinds of phenomena (objects and processes) and explain the origin of new kinds. As we extend this search to consider the origin of human mental phenomena and human culture, science recursively comes to study itself, promising a kind of unification (or at least closure) of the sciences. This advancement rests on a number of assumptions, such as the existence of different kinds of phenomena and our ability to construct good theories involving multiple kinds of phenomena.

Different kinds of phenomena are believed to exist, and are described by biological, psychological, social, physical, and mechanical systems (to name a few). But satisfaction of the second assumption is perhaps less obvious. One of the necessary conditions for having a theory is the existence of observables, usually measured quantities, in terms of which predictions of the theory can be made and tested. It then follows that progress in transdisciplinary science requires observables that can be taken on more than one kind of phenomenon. Clearly some measurements can be taken on more than one kind of thing, but not all kinds of things. For example, both houses and cats can be massed, but songs cannot; both societies and ecologies have growth rates, but isotopes do not. Other quantities are unique to objects of specific kinds: Only national economies have balances of trade.

### The Disparate Uses of Entropy

But for the latter half of this century, entropy has been ubiquitously offered as being applicable to systems of all kinds. Many researchers have defined nonthermodynamic entropies or "entropy analogs" to be used in theoretical descriptions of systems other than purely physical or informational. Entropic terms are used in the analysis of physical structures (Rivier 1986, Weaire and Rivier 1984), biological systems (Brooks and Wiley 1988, Conrad 1983, Gatlin 1972, Schneider 1988), social and economic systems (Bailey 1990, Batten 1983, Georgescu-Roegen 1971, Hershey and Lee 1987, Klapp 1975, McFarland 1969, Theil 1967), formal systems (Adler and Konheim 1965, Schuster 1984), and even moral theory (Galtung 1975, Peacocke 1984).

The use of entropy terms and a host of related terms (e.g., order, complexity, organization, information, uncertainty, randomness, and variety) have penetrated virtually every area of scientific inquiry, to the point where some see them as having universal explanatory power (Georgescu-Roegen

1971, Rifkin 1980). Many natural philosophers, when writing about universal evolution, seem compelled to at least discuss the concept (Davies 1988, de Chardin 1959, Laszlo 1987). Other transdisciplinary scientists seem to be approaching a consistent theory of global evolution using entropic terms (Georgescu-Roegen 1971, Rifkin 1980). While the more universalist claims are not generally held in high regard (Pagels 1985, Sibatani 1982), nevertheless this "entropy phenomena" deserves serious attention.

In considering these disparate uses of entropy and nonthermodynamic entropies, two questions come to mind. The first is whether it is "legitimate" to use nonthermodynamic entropies in these biological, ecological, economic, and social theories. The other is, assuming this legitimacy is established, what is the significance for the kind of scientific unification described above? Where one stands on the first question will depend on one's understanding of the issues discussed above, and, in particular, on how one defines entropy, either the statistical (syntactic) sense or as a measured quantity in some specific domain (semantic).

## The Risks of Narrow Interpretation

In the recent debate on the significance of thermodynamics for biology (Bookstein 1983, Brooks and Wiley 1988, Lovtrup 1983, Morowitz 1986, Wicken 1983, Wiley and Brooks 1982), there are those who take the latter position. They argue, from historical grounds, that the original thermodynamicists were working on a purely empirical basis. Since, for a time, entropy was defined only as a measured quantity on thermodynamic systems, extensions of this concept are meaningless. Or, since the essential property of thermodynamic entropy is its monotonic increase in isolated systems, and since it is not clear that nonthermodynamic entropies have such a property, they therefore lack an essential property of any entropy. On this view, clearly only thermodynamic entropies are "legitimate." However, if we accept the challenge of the Shannon-Jaynes program to extend the entropy concept, then any system that is describable in stochastic, probabilistic terms is necessarily also describable in terms of a (presumably nonthermodynamic) entropy. Then, as we consider our second question, we have to see that this quantity might not display a monotonic increase in a closed system according to some analog of the second law, thus threatening the promised scientific unification. But on the Shannon-Jaynes view, this result seems inevitable.

In fact, we must admit that such measures of nonthermodynamic entropies might not be theoretically useful at all: a quantity simply being definable or measurable on a system does not entail that it is significant for some purpose. Of course, this quantity may, in fact, be used in a second law correlate, or in some other nature law. But if this comes about, it will be an interesting a posteriori, theoretical development, not a simple truth following from the "invocation" of an entropy.

However, if we do not take the Shannon–Jaynes view, then entropy can never be used in the kind of transdisciplinary theorizing that systems research intends. To do so, it is necessary to create theories that involve multiple kinds of phenomena and observations that are qualitatively distinct from the purely physical. If we restrict ourselves to thermodynamic entropies, then we are always working at the thermodynamic (physical) level of description, never in, for example, biological, psychological, sociological, or symbolic terms. It would remain for some other measure to serve that crucial role.

### The Extension of the Thermodynamic Program

Now it is true that there is a renewed interest in the use of thermodynamic principles, the second law, and thermodynamic entropy in the explanation of the development of complex systems and emergent phenomena. It is certainly interesting and important to measure the thermodynamic properties of different kinds of systems (e.g., in global ecology (Sibatani 1982, Ulanowicz and Hannon 1987) or in ecological succession (Schneider 1988)).

But these programs are not necessarily in a transdisciplinary spirit. Thermodynamic theory deals solely with physical quantities, and does not involve measurements or observations made on other kinds of phenomena in systems. Like the "biomass" concept, they are simply extensions of a physical quantity (in this case entropy) to systems that happen to involve other kinds of processes as well. Nor does the success of efforts to extend the usefulness of thermodynamic entropy entail the failure of the use of nonthermodynamic entropies in other contexts. These are distinct research programs, which must be judged on their own merits. It remains to be seen whether nonthermodynamic entropies are useful in these other contexts or not.

## AN EXAMPLE: BÈNARD CONVECTION

As an illustration of what is at stake in this argument, consider a simple dissipative structure: convection in Bènard cells. This system has been deeply analyzed. In terms of thermodynamics, it arises as the first symmetry-breaking event in the move off the "thermodynamic branch" and away from equilibrium (Nicolis and Prigogine 1977). In dynamical systems theory, it is the most studied and celebrated of the "strange attractors," and is characterized by the Lorenz equations and the Lorenz attractor (Curry and Herring 1984, Lorenz 1963). It arises in a simple three-dimensional nonlinear differential system due to a Hopf bifurcation (Beltrami 1987, pp. 217–218).

Swenson (1989) has offered Bènard convection as an exemplar of the simplest self-organizing system, and has proposed a law of maximum entropy production. This law should hold far from equilibrium, and stands in sharp contrast to Prigogine's theorem of minimum entropy production, which holds close to equilibrium (Nicolis and Prigogine 1977). Swenson suggests that the proposed law has significant implications for general evolutionary theory.

We will not discuss the experiment in depth, except to note that an "order parameter" is established as a thermal energy gradient across a fluid in an experimental apparatus. As the parameter is moved past a bifurcation point, the flow changes from conductive, linear flow to convective, nonlinear flow, and convection cells appear. It can be shown that beyond the bifurcation and for a constant gradient, the number, size, and shape of the convection cells will approach a steady state that is characterized by cells of uniform size and shape packed in a hexagonal lattice, and that maximizes the dissipation of the overall thermodynamic system.

## Characterization of Emergent Entities

We wish to regard these convection cells as examples of emergent phenomena, and say that beyond the critical point entities at two levels of analysis, the convection cells and the liquid itself, exist simultaneously. Further, we wish to be able to apply an entropy measure to each kind of thing separately. We agree with the spirit of Swenson's work, which holds that until and unless these simple laboratory examples are clearly addressed, more ambitious attempts at the level of biological and social objects will not be well founded.

In our example, then, let us be very careful to define exactly what these levels of analysis are. In particular, define level zero entities as $E_0$, the molecules of the liquid; and define level two entities as $E_2$, the whole volume of liquid in the experimental apparatus (we will not define level one for the moment).

It is clear that before the bifurcation, both the measured and statistical entropy of the liquid $E_2$ is well defined by equations 1 and 2 as the relation from the set of mechanical states of the molecules $E_0$ to the set of thermodynamic states of the liquid $E_2$. After the bifurcation, we recognize the existence of convection cells because they manifest phenomena outside of the spatial and temporal scales of both levels zero and two. We recognize relatively discontinuous natural boundaries between these entities, and thus designate them as level one entities $E_1$.[3]

## Type Specificity and the Emergence of Levels of Analysis

A statistical entropy measure is defined by a many-to-one relation. Since we have three kinds of things, we now have three choices for a possible entropy measure: from level zero to level one, the entropy of the convection cells "themselves"; from level one to level two, the entropy of the whole liquid *defined at the level of the convection cells*; and finally from level zero to level two, the entropy of the whole liquid *defined at the level of the molecules*. Denote these quantities as $S_{0,1}$, $S_{1,2}$, and $S_{0,2}$, respectively.

Now, how do we understand these possible measures and the relation between them?

- The phase space for $S_{0,2}$ does not change through the bifurcation. The individual molecules can move over the same volume and can have the same velocities as before. Since the molecular velocities are now constrained by being taken up by convection cycles, we can see that the number of mechanical states accessible by them is also reduced. Therefore, the entropy $S_{0,2}$ is reduced while entropy production to the environment is increased. This process is in keeping with the second law of thermodynamics and the traditional analysis of self-organization (Nicolis and Prigogine 1977, von Foerster 1960).

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[3] We usually think about emergent phenomena as entering at the "pinnacle" of some evolutionary hierarchy. In contrast, we regard this kind of emergence not as adding a layer on top of a hierarchy, but rather as a "deepening" or "infolding" of an existing hierarchy. The new level is added between what was previously (and still is) the bottom and top levels. This explains our choices for labeling levels. The temporal ordering of level one following levels zero and two is not equivalent to the structural and scale ordering from levels zero to one to two (Salthe 1985).

- After the bifurcation, the quantity $S_{0,2}$ can be measured in exactly the same way as before, in accordance with equation 1, through measurements of the temperature and heat flow.
- The quantity $S_{0,1}$ (the entropy of the convection cells "themselves") is defined as if each cell was a separate thermodynamic system in and of itself. Inside the boundaries of the cell walls, the temperature and heat flow is well defined.

What is the quantity $S_{1,2}$, the entropy of the whole liquid defined *in terms of the cells*? Should it, perhaps, be defined as the sum, or some other function, of the cell thermodynamic entropies $S_{0,1}$? I would suggest not. We wish to regard the convection cells as objects with some autonomy and identity, and define the $S_{1,2}$ quantity over a *new* phase space at a *qualitatively different level of analysis* from the thermodynamic level. What is required is the recognition of properties inherent to the convection cells themselves, such as their number, their energies, positions, sizes, shapes, relative velocities, etc.

The process of recognizing emergent levels is necessarily scale-dependent. Denote the number of elements $E_i$ in existence at a level i as $n_i$, $i \in \{0, 1, 2\}$. What is then crucial about the convection experiment is that $\log(n_0) \gg \log(n_1) \gg \log(n_2)$, where $n_2$, the number of experimental aparati, is one. For example, in our case, $\log(n_0) \approx 28$, as the system approaches steady state $\log(n_1) \approx 2$, and $\log(n_2) = 0$.

This property alone allows the recognition of new levels. When this property does not hold, we cannot recognize new levels. For example, when $n_1 = 1$ there is exactly one convection cell, and the liquid rotates as a whole. In this case, we describe the convection only as *a property of the liquid* $E_2$, and not as an *entity* $E_1$. Similarly, it would not be possible to define an entropy $S_{1,2}$ of the liquid in terms of the cells, because with only one cell there is a "degenerate" distribution of cell properties, with only one element. Whenever $n_1$ is small, say $n_1 = 2$, there is not a sufficient sample size to define a valid probability distribution of properties of the cells, and therefore it is not possible to go on to derive $S_{1,2}$.

But given the existence of new entities with some properties, and assuming that a valid probability distribution can be derived for these properties, then a nonthermodynamic entropy for $S_{1,2}$ will be available. It should be clear that the basis for making such an identification will be the statistical (syntactic) one, where satisfaction of the axioms necessary for statistical entropy is maintained. In fact, our terminology is incorrect, because the use of $S_{1,2}$ indicates a thermodynamic quantity. Instead, we should use $H_{1,2}$ to indicate that the entropy defined on the cells is a purely statistical, nonthermodynamic entropy. Thus, we need not look for correlates to temperature, pressure, and energy at the level of the convection cells, and no formulation like the second law need *necessarily* hold at the level of the cells.

### Emergence and Dynamic Equilibrium

We noted above that, over a relatively long time scale, the cells approach a steady state of hexagonal packing, which can be shown to maximize their energy flow and thermodynamic entropy production. Since we recognize these entities at their own level of analysis, it is possible to consider that this is truly a state of equilibrium *at the level of the convection cells.* In fact, it is a *dynamic* equilibrium, dependent on the constant ongoing activity and flow at the level below.

Indeed, we can go so far as to say that the critical difference between static and dynamic equilibria is the emergence of qualitatively distinct levels of analysis. Steady states in flow systems and stable cyclic or chaotic attractors of dynamical systems are exactly such kinds of dynamic equilibria. They are dynamic in that they are dependent on the long-run statistical stability of an underlying process, which is ongoing at a faster time scale and exactly *out of* equilibrium. Thus, the experimental apparatus defined in terms of molecules is out of thermodynamic equilibrium, but after sufficient time is in a nonthermodynamic equilibrium at the level of the convection cells.

## FUTURE DIRECTIONS IN MULTILEVEL STUDIES

Issues of processes at multiple temporal and spatial scaling are being examined (Auger 1990, Minch 1988, Ruelle 1987), and underlie the extensions of dynamic systems theory into catastrophe theory (Thom 1983, Zeeman 1977). But it should be noted that such phenomena are quite common. Indeed, as we have seen, the simple "emergence" of macroscopic properties of substances from the quantum-mechanical actions of their constituent molecules is a perfect example. Thus, we have to recognize emergent phenomena as being quite ordinary. Rather than some "creative" aspect of evolution that involves seemingly esoteric states of nature as far-from-equilibrium thermodynamics, biological evolution, or the human mind, emergence is a fundamental physical process at all levels of analysis.

We get some indication of the direction that might be taken from some recent work on the distinctions between different types of entropies. For example, we now understand (Frautschi 1988, Hawking 1976) that a monatomic gas at thermal equilibrium does not in fact have maximal entropy, because of gravitational forces between the molecules. In a gas of sufficient size, these forces become significant, resulting in gravitational collapse, eventually leading to a singularity. The entropies of black holes can be calculated and measured, and in fact this is a more stable state for the gas than its noncollapsed form.

Thus we see that the overall entropy of this system has contributions from multiple components: in this case, thermal and gravitational entropy. In a similar way, physicists define a great array of types of entropies: nuclear, chemical, thermal, gravitational, positional, and entropies of mixing. Gibbs' paradox (Denbigh and Denbigh 1985, Chapter 4), when described in terms of nuclear isotopes, can be seen as a problem of sorting out nuclear from chemical entropy. Denbigh and Denbigh note that traditional solutions to Gibbs' paradox address the issue not as originally posed by Gibbs, as a problem of macroscopic thermodynamics, but rather as a problem of macroscopic statistical physics.

## CONCLUSION

By extension, we should be able to define entropies at other levels: for example, genetic, ecological, economic, or social. What are the restrictions on this process? It seems that we are in a clear position to say. In particular, all that is necessary is the identification of type-specific states of nature that conform to the axioms necessary for entropy measures. Of course, the results from the developments in thermodynamics will not necessarily follow to these new usages, but they may do so contingently. But to the extent that the concepts of order, organization, complexity, information, uncertainty, randomness, and variety are related to entropy measures, they then might apply to nonthermodynamic entropies as well. And perhaps new type-specific laws of nature at these new levels, or relations between these types and those at the thermodynamic levels, will also be forthcoming.

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