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Strong Probabilistic Compatibility of Possibilistic Histograms*

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Abstract

Some properties of empirical random sets and possibilistic histograms related to strong probabilistic compatibility are described. We will discuss possibilistic histograms and the possibility of occurence, the nature of probability distributions which are strongly stochastically compatible with a given possibility distribution, and the derivation of frequency distributions from empirical random sets.

1 Introduction

Possibility theory [3] is an alternative information theory to that based on probability. Although possibility theory is logically independent of probability theory, they are related: both arise in Dempster-Shafer evidence theory as fuzzy measures defined on random sets; and their distributions are both fuzzy sets. So possibility theory is a component of a broader Generalized Information Theory (GIT), which includes all of these fields [12].

Zadeh's concept of probabilistic-possibilistic consistency [16] is an example of the kind of principle which can be brought to bear on the problem of deriving a coherent, synthetic GIT. In order to accommodate the desired properties of possibilistic semantics, Joslyn has extended this idea to a principle of strong compatibility (or consistency) [7].

Another example of a synthetic principle is the use of random sets—originally developed as a branch of stochastic geometry [10]—to provide a broad, unifying context within which to develop GIT [4]. Joslyn has also used random sets to ground possibility theory on an *empirical* basis by developing methods for the measurement of possibility distributions, and in particular possibilistic histograms, based on empirical random sets, in turn derived from the collection of set-valued observations [5, 8].

Some properties of empirical random sets and possibilistic histograms are described related to strong probabilistic compatibility. After introducing the fundamentals of possibilistic mathematics and measurement, we will discuss possibilistic histograms and the possibility of occurence, the nature of probability distributions which are strongly stochastically compatible with a given possibility distribution, and the derivation of frequency distributions from empirical random sets.

2 Mathematical Preliminaries

Assume a finite universe $\Omega := \{\omega_i\}, 1 \le i \le n$.

2.1 Possibilistic Mathematics

The function $m\colon 2^\Omega \mapsto [0,1]$ is an **evidence function** (otherwise known as a **basic probability assignment**) when $m(\emptyset) = 0$ and $\sum_{A \subseteq \Omega} m(A) = 1$. Denote a **random set** generated from an evidence function as

$$\mathcal{S} := \{\langle A_j, m_j \rangle : m_j > 0\}, \qquad (1)$$

where $\langle\cdot\rangle$ is a vector, $A_j \subseteq \Omega$, $m_j := m(A_j)$, and

$$1 \le j \le N := |\mathcal{S}| \le 2^n - 1.$$

Denote the **focal set** of $\mathcal{S}$ as $\mathcal{F} := \{A_j : m_j > 0\}$. $\mathcal{S}$ has **core** and **support**

$$\mathbf{C}(\mathcal{S}) := \bigcap_{A_j \in \mathcal{F}} A_j, \qquad \mathbf{U}(\mathcal{S}) := \bigcup_{A_j \in \mathcal{F}} A_j$$

respectively, and is **consistent** if $\mathbf{C}(\mathcal{S}) \ne \emptyset$.

The **plausibility** and **belief** measures on $\forall A \subseteq \Omega$ are

$$\mathrm{Pl}(A) := \sum_{A_j \not\perp A} m_j, \qquad \mathrm{Bel}(A) := \sum_{A_j \subseteq A} m_j,$$

where $A - B$ denotes $A \cap B = \emptyset$. Pl and Bel are generally non-additive fuzzy measures [15], and are dual,

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in that $\forall A \subseteq \Omega, \mathrm{Bel}(A) = 1 - \mathrm{Pl}(\overline{A})$. In general only plausibility will be considered below. The **plausibility assignment** (otherwise known as the **one-point coverage function**) of $\mathcal{S}$ is $\vec{\mathrm{Pl}} = \langle \mathrm{Pl}_i \rangle := \langle \mathrm{Pl}(\{\omega_i\}) \rangle$, where

$$\mathrm{Pl}_i := \sum_{A_j \ni \omega_i} m_j.$$

When $\forall A_j \in \mathcal{F}, |A_j| = 1$, then $\mathcal{S}$ is **specific**, and $\Pr(A) := \mathrm{Pl}(A) = \mathrm{Bel}(A)$ is an additive **probability measure** with

$$\forall A, B \subseteq \Omega, \quad \Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B). \quad (2)$$

Then $\vec{p} = \langle p_i \rangle := \vec{\mathrm{Pl}}$ is a **probability distribution** with additive normalization and operator

$$\sum_i p_i = 1, \qquad \Pr(A) = \sum_{\omega_i \in A} p_i.$$

It is well known that statistical entropy is the canonical measure of information in probability theory [13]. In general, the probability distribution with maximal entropy is the **maximally uninformative probability distribution** denoted $\vec{p}^*$, and results when $\forall i, p_i = 1/n$ [9]. Given a random set $\mathcal{S}$, then the Maximum Entropy Principle (MEP) [12] has been applied [2] to derive a canonical probability distribution $p^{\mathcal{S}}$ approximating $\mathcal{S}$, replacing each subset evidence value $m(A_j)$ with the MEP uniform probability distribution over its members, so that

$$\forall \omega \in \Omega, \qquad p^{\mathcal{S}}(\omega) := \sum_{A_j \ni \omega} \frac{m_j}{|A_j|}. \quad (3)$$

$\mathcal{S}$ is **consonant** ($\mathcal{F}$ is a **nest**) when (without loss of generality for ordering, and letting $A_0 := \emptyset$) $A_{j\perp 1} \subseteq A_j$. Now $\Pi(A) := \mathrm{Pl}(A)$ is a **possibility measure** and $\eta(A) := \mathrm{Bel}(A)$ is a **necessity measure**.[1] As Pr is additive, so Π is **maximal**:

$$\forall A, B \subseteq \Omega, \quad \Pi(A \cup B) = \Pi(A) \vee \Pi(B),$$

where $\vee$ is the maximum operator. $\vec{\pi} = \langle \pi_i \rangle := \vec{\mathrm{Pl}}$ is now a **possibility distribution** with maximal normalization

$$\bigvee_i \pi_i = 1 \quad (4)$$

and operator

$$\Pi(A) = \bigvee_{\omega_i \in A} \pi_i. \quad (5)$$

[1] Since results for necessity are dual to those of possibility, only possibility will be discussed in the sequel.

The **maximally uninformative possibility distribution** denoted $\vec{\pi}^*$, has maximal nonspecificity [12], and results when $\forall i, \pi_i = 1$ [9].

The condition (4) for $\vec{\mathrm{Pl}}$ to be a possibility distribution is actually achieved whenever $\mathcal{S}$ is even consistent (which is required when $\mathcal{S}$ is consonant). When $\mathcal{S}$ is consistent but not consonant, then even though $\vec{\mathrm{Pl}}$ is a possibility distribution by (4), Pl is not a possibility measure Π. Then there is a unique possibilistic approximation Π^* to Pl achieved by invoking (5) on π [9]. If $\mathcal{S}$ is not even consistent, then there are well-justified possibilistic normalization methods [6, 14] such as "consistent transformations" [6], which select certain elements or regions of π to be "elevated" to be in a core.

The following result will be useful below:

Corollary 6 If $\mathcal{S}$ is consistent, then $m(A) > 0 \rightarrow \mathrm{Pl}(A) = 1$.

Proof: Fix $A \subseteq \Omega$. Since $\mathcal{S}$ is consistent, $\mathbf{C}(\mathcal{S}) = \bigcap_{A_j \in \mathcal{F}} A_j \neq \emptyset$, so that $\forall A_{j_1}, A_{j_2} \in \mathcal{F}, A_{j_1} \not\perp A_{j_2}$. Since $m(A) > 0$, therefore $A \in \mathcal{F}$, and so $\forall A_j \in \mathcal{F}, A \not\perp A_j$. Therefore $\mathrm{Pl}(A) = \sum_{A_j \not\perp A} m_j = \sum_{A_j \in \mathcal{F}} m_j = 1$. ∎

2.2 Possibilistic-Probabilistic Compatibility

This paper concerns situations where probability and possibility are considered together. Measures of **compatibility**[2]between a probability and a possibility distribution are available [1]. The best known of these is Zadeh's measure [16]

$$\gamma_Z(\vec{p}, \vec{\pi}) := \vec{p} \cdot \vec{\pi} = \sum_{i=1}^n p_i \pi_i,$$

where $\gamma_Z(p, \pi) = 1$ indicates maximal compatibility and $\gamma_Z(p, \pi) = \bigwedge_i \pi_i$ minimal compatibility, and $\wedge$ is the minimum operator.

We now introduce some ideas from probabilistic measurement. Assume a counting function $c: \Omega \mapsto \mathcal{W}$ such that $c_i := c(\omega_i)$ is the count of the occurrences of ω_i in a statistical record. Then a **frequency distribution** is a function $f: \Omega \mapsto [0, 1]$ where

$$f(\omega_i) = f_i := \frac{c_i}{\sum_i c_i}.$$

Denote the vector $\vec{f} := \langle f_i \rangle$. The **frequency measure** is a function $P: 2^{\Omega} \mapsto [0, 1]$ where $\forall A \subseteq \Omega$,

$$P(A) := \sum_{\omega_i \in A} f_i.$$

[2] The term used in the literature is actually "consistency", so to avoid confusion with random set consistency, we will use "compatibility".

$\vec{f}$ is a natural probability distribution with normalization $\sum_i f_i = 1$, and P is a natural probability measure as in (2).

Many methods are available to convert a given probability distribution to a possibility distribution, and vice versa [13]. One of the most prominent is the maximum normalization or ratio scale method [11]. Given a frequency distribution f, then let $\pi^m : \Omega \mapsto [0,1]$ be a possibility distribution where

$$\pi^m(\omega_i) = \pi_i^m := \frac{c_i}{\bigvee_i c_i}.$$

It follows [9] that

$$\pi_i^m = \frac{f_i}{\bigvee f_i}, \qquad f_i = \frac{\pi_i^m}{\sum \pi_i^m}. \tag{7}$$

We also have the following result [9].

Proposition 8 If $\gamma_Z(f, \pi^m) = 1$ then $\vec{f} = \vec{p}^*$ and $\vec{\pi}^m = \vec{\pi}^*$.

Joslyn has also considered the semantics of possibility theory from a number of different perspectives, including the contexts of graduated, physical, and modal conceptual frameworks [9]. In particular, he has considered what an appropriate relation between probabilistic and possibilistic representations of the same problem domain would be [7], and has asserted the following strong principle.

Principle 9 (Strong Probability-Possibility Compatibility (PPC)) For a given probability measure Pr and possibility measure Π to be strongly compatbile, then

$$\forall A \subseteq \Omega, \quad \Pr(A) > 0 \leftrightarrow \Pi(A).$$

It follows that $\Pr(A) = 0 \leftrightarrow \Pi(A) < 1$, and

$$\eta(A) > 0 \rightarrow \Pr(A) > 0, \qquad \eta(A) = 0 \rightarrow \Pr(A) = 0.$$

At the distribution level it follows that

$$\forall \omega \in \Omega, \quad p(\omega) > 0 \leftrightarrow \pi(\omega) = 1, \quad p(\omega) = 0 \leftrightarrow \pi(\omega) < 1.$$

Finally, if the distributions $\vec{p}$ and $\vec{\pi}$ are strongly compatibile, then $\gamma_Z(\vec{p}, \vec{\pi}) = 1$.

Note that the PPC (9) is not a definition or a theorem, but is rather a *principle* asserted as a *semantic* criterion, and is thus necessarily extra-theoretical. Detailed arguments justifying this position are offered elsewhere [7, 9]. Suffice it here to say that the PPC states that something having non-zero probability is *likely*, and therefore given sufficient time *eventual*, and therefore equivalent to its being *completely possible*. Conversely, a properly possible event $(0 < \Pi(A) < 1)$ must be of probability measure zero, and probability zero may or may not indicate proper possibility.

2.3 Possibilistic Measurement

Measurement methods for possibility distributions have developed by Joslyn [5, 8, 9]. To derive a possibility distribution from an empirical source, it is necessary to observe subsets $B_s \subseteq \Omega, 1 \leq s \leq M$ denoted as a vector $\vec{B} := \langle B_s \rangle$. The set of observed subsets produced by eliminating any duplicates in $\vec{B}$ is an **empirical focal set** $\mathcal{F}^E := \{A_j\}$, where $N \leq M$ and $\forall A_j \in \mathcal{F}^E, \exists B_s \in \vec{B}, B_s = A_j$, and inclusion of an element in a vector is defined as appropriate.

Denoting the number of times that a given set $B_s = A_j$ occurs in $\vec{B}$ as $C(A_j)$, then the **set-frequency function** is

$$m^E : \mathcal{F}^E \mapsto [0,1], \qquad m^E(A_j) = \frac{C_j}{\sum_{A_j \in \mathcal{F}^E} C_j} = C_j / M, \tag{10}$$

where $C_j := C(A_j)$. m^E is clearly an evidence function, which in turn generates an **empirical random set** denoted $\mathcal{S}^E$ according to (1). If $\mathcal{F}^E$ is a disjoint class, then $\mathcal{S}^E$ generates a probability distribution on an equivalence class on $\mathbf{U}(\mathcal{F})$. But if $\mathcal{S}^E$ is consistent, then the **empirical possibility distribution** is

$$\pi(\omega) = \sum_{A_j \ni \omega} m_j^E = \frac{\sum_{A_j \ni \omega} C_j}{M}. \tag{11}$$

Possibility distributions derived according to (11) can be properly described as **possibilistic histograms**, similar to ordinary (stochastic) histograms, but generated from possibly overlapping interval observations, and thus governed by the mathematics of random sets. In the sequel it will be assumed that $\mathcal{S}^E$ is consistent, either naturally or as the result of a normalization method, and thus π from (11) is a possibility distribution.

An example is shown in Fig. 1. On the left, four observed intervals are shown. The bottom two occur with frequency 1/2, while each of the upper two have frequency 1/4. Together they determine $\mathcal{S}^E$. The step function on the right is the possibilistic histogram π derived from (11). It can be briefly stated in vector form as

$$\vec{\pi} = \langle 1/4, 1, 1/2, 1/4 \rangle$$

where the values are taken on each of the piecewise constant segments

$$\langle [1, 1.5), [1.5, 2], (2, 3.5], (3.5, 4] \rangle, \tag{12}$$

of the step function, as shown in the figure. Also shown in the figure are two examples of the variety of well-justified continuous approximations to a possibilistic histogram [8]. This approach to possibilistic measurement generalizes to n intervals and to the continuous case.

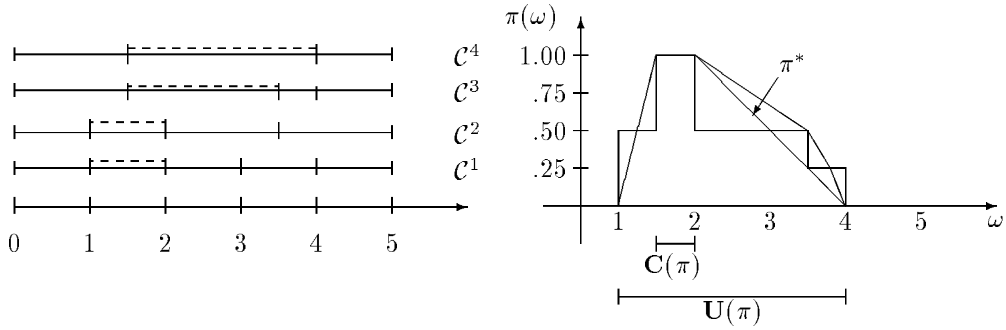

Figure 1: (Left) Four example observed intervals. (Right) The possibilistic histogram and two continuous approximations.

3 Strongly Compatible Probability Distributions

From the natural language perspective, the primary semantic criterion for possibility is that the occurrence of an event requires maximal (unitary) possibility. In a possibilistic histogram the occurring events are exactly those $B_s \in \vec{B}$ which have been observed. So this condition is easily met by possibilistic histograms.

Corollary 13 If $\mathcal{F}^E$ is consistent, then $\forall B_s \in \vec{B}, \Pi(B_s) = 1$.

Proof: Fix B_s. Then $C(B_s) \geq 1$, so $m(B_s) \geq 1/M > 0$. The result follows from the corrolary (6) and the consistency of $\mathcal{F}^E$. ■

Probability distributions which conform to the PPC with a possibilistic histogram should also be considered. Under the PPC (9), it is necessary that $p(\omega) = 0$ wherever $\pi(\omega) < 1$, that is $\forall \omega \notin \mathbf{C}(\mathcal{F}^E)$. In the example in Fig. 1, that would yield $p > 0$ only on the interval $\mathbf{C}(\pi) = [1.5, 2)$. No further information would be provided by π, and so the MEP would yield the uniform probability density

$$p^*(\omega) = \begin{cases} 2, & \omega \in [1.5, 2) \\ 0, & \text{elsewhere} \end{cases} .$$

This result makes complete sense in the context of the nature of subset measurements. Given a consistent set of observed intervals, if they are all to be believed then all that can be said is that the event actually happened *somewhere* in the core. There the possibility is unitary, and by the PPC the probability is positive. But there is no further information about the *likelihood* of the event being anywhere *particular* inside the core, thus requiring the maximally uninformative probability distribution p^*.

The fact that $\forall \omega \in \mathbf{U}(\pi), \omega \notin \mathbf{C}(\pi), 0 < \pi < 1$ indicates that it is *somewhat* possible for *another* observation, perhaps at another time, to be found somewhere between the core and the edge of the support, but not *completely* possible, since nothing can be said to have been actually observed there yet. Thus the subset measurements give *no* likelihood information about the occurrence of an ω in this region, and by the PPC $p = 0$ there.

If $\mathcal{S}^E$ is inconsistent, and thus a consistent approximation must be made, then for a focus $\omega_0 \in \Omega, \mathbf{C}(\mathcal{S}^E) = \{\omega_0\}$, and so p will be a Dirac-delta function at ω_0.

4 Frequency Distributions from Empirical Random Sets

It is also interesting to see how a purely "probabilistic" treatment would approach set-statistics. In particular, it is possible to use other counting methods to derive an ordinary frequency distribution $\vec{f}$ from the counts attached to each observed subset.

4.1 Frequency Analysis of Subset Measurements

In order to simplify the problem, consider the case of two overlapping observations on a discrete universe. Let $\Omega = \{a, b, c\}$, and assume two observations $B_1 = \{a, b\}$ and $B_2 = \{b, c\}$, so that $C(B_1) = C(B_2) = 1$.

On a pure frequency analysis at the level of the subsets B_s, then $\Pr(B_1) = \Pr(B_2) = 1/2$. Under the assumption that Pr should have an additive probability distribution $p \colon \Omega \mapsto [0, 1]$, then

$$\begin{array}{ccccccc} p(a) & + & p(b) & + & p(c) & = & 1 \\ p(a) & + & p(b) & & & = & 1/2 \\ & & p(b) & + & p(c) & = & 1/2 \end{array}$$

which has the solution $p(a) = p(c) = 1/2, p(b) = 0$. This is entirely unsatisfactory, and maximally incompatible with the possibilistic results above: it *eliminates* probability exactly on b, the point where there is the *most* evidence, and where in the possibilistic histogram $\pi(b) = 1$.

Only slightly more complicated cases, such as the example in Fig. 1, reveal that this method frequently does not yield *any* feasible solutions for non-negative probabilities.

4.2 Subset to Element Counts

Another approach is to translate the counts on subsets into counts on elements, thus establishing a mapping $C \mapsto c$. There are a number of ways in which that could be done.

4.2.1 Duplicated Counts

We could say that a nonspecific observation is really an observation of *every* element of the subset. Then each observation of a subset B_s would contribute one element count for every $\omega \in B_s$. Then the overall element count is

$$\forall \omega \in \Omega, \quad c(\omega) = \sum_{A_j \ni \omega} C_j. \qquad (14)$$

Corollary 15

$$f(\omega) = \frac{c(\omega)}{\sum_{A_j \in \mathcal{F}^E} C_j |A_j|}.$$

Proof:

$$f(\omega) = \frac{c(\omega)}{\sum_{\omega\in\Omega} c(\omega)} = \frac{c(\omega)}{\sum_{\omega\in\Omega}\sum_{A_j \ni \omega} C_j} = \frac{c(\omega)}{\sum_{A_j \in \mathcal{F}^E} C_j |A_j|}.$$

■

By this method, the example in Fig. 1 yields the frequency distribution

$$\vec{f} = \langle 2/9, 4/9, 2/9, 1/9 \rangle .$$

similarly valued over the piecewise constant segments in (12). Note that this is identical to $\vec{\pi}$ for elements having the same numerator, but the denominator changed from 4 (which is $\sum C_j$) to 9 (which is $\sum c(\omega) = \sum C_j |A_j|$).

In fact, the effect of this count duplication method is to establish a maximum normalized ratio scale between π and f.

Theorem 16 Given a consistent $\mathcal{F}^E$ with a frequency distribution f determined by (14), then $\forall \omega \in \Omega$,

$$f(\omega) = \frac{\pi(\omega)}{\sum \pi(\omega)}, \qquad \pi(\omega) = \frac{f(\omega)}{\bigvee f(\omega)}$$

Proof: From the possibilistic histogram formula (11) and (14),

$$\forall \omega \in \Omega, \quad M\pi(\omega) = \sum_{A_j \ni \omega} C_j = c(\omega).$$

Therefore from the corollary (15),

$$f(\omega) = \frac{c(\omega)}{\sum_{\omega\in\Omega} c(\omega)} = \frac{M\pi(\omega)}{\sum_{\omega\in\Omega} M\pi(\omega)} = \frac{\pi(\omega)}{\sum_{\omega\in\Omega}\pi(\omega)}.$$

The second result follows from the ratio scale frequency conversion (7). ■

Thus the disadvantages of duplicating counts like this are clear. First, frequency additivity is violated because

$$\sum_{\omega \in A_j} c(\omega) = \sum_{\omega\in A_j}\sum_{A_k \ni w} C_k \geq C_j.$$

Also, the PPC is generally violated in virtue of the ratio scale frequency conversion, as shown in Prop. (8).

4.2.2 Distributed Counts

Instead of a subset count contributing multiple element counts, the single subset count can be additively distributed amongst the $\omega \in A$. Since there is no further information about how to distribute the count, then by the MEP a uniform distribution should be used. Then the element count for each $\omega \in \Omega$ is

$$\forall \omega \in \Omega, \quad c(\omega) = \sum_{A_j \ni \omega} \frac{C_j}{|A_j|}. \qquad (17)$$

Corollary 18 $f(\omega) = c(\omega)/M$.

Proof: Because

$$\sum_{\omega\in\Omega} c(\omega) = \sum_{\omega\in\Omega}\sum_{A_j\ni\omega}\frac{C_j}{|A_j|} = \sum_{A_j\in\mathcal{F}^E}\frac{C_j|A_j|}{|A_j|} = \sum_{A_j\in\mathcal{F}^E} C_j = M,$$

therefore

$$f(\omega) = \frac{c(\omega)}{\sum_{\omega\in\Omega} c(\omega)} = \frac{c(\omega)}{M}.$$

■

By this method, the example in Fig. 1 yields a frequency distribution

$$\vec{f} = \langle 1/4, 11/24, 5/24, 1/12 \rangle .$$

Not surprisingly, this method is closely related to the applications of the MEP as discussed above.

Theorem 19 Assume an empirical random set $\mathcal{S}^E$ and let f be a frequency distribution determined by (17). Then f is the maximum entropy probability distribution $p^{\mathcal{S}^E}$ from (3).

Proof: From (17), (18), the set-frequency definition (10), and the maximum entropy probability distribution formula (3), then $\forall \omega \in \Omega$,

$$f(\omega) = \frac{c(\omega)}{M} = \sum_{A_j \ni \omega} \frac{C_j}{|A_j| M} = \sum_{A_j \ni \omega} \frac{m_j^E}{|A_j|} = p^{\mathcal{S}^E}(\omega).$$

■

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