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The Process Theoretical Approach to Qualitative DEVS*

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Abstract

Qualitative extensions to the Discrete EVent Systems (DEVS) formalism are presented based on some new ideas relating general process and automata theory and General Information Theory (GIT). Specifically, we will consider the applicability of possibilistic and fuzzy processes and automata to the DEVS methodology. Much of the DEVS formalism can be cast into classical finite automata theoretical terms. Classical automata generalize to qualitative temporal processes with input and output where states are valued on a lattice and the state transition function is implemented by algebraic semiring operators. Special cases include classical deterministic, nondeterministic, and stochastic (Markov) processes, and the newer fuzzy and possibilistic processes. In this way DEVS can be extended qualitatively in terms of general Moore automata.

1 Introduction

The Discrete EVent Systems (DEVS) methodology [30] has emerged in recent years as an important modeling and simulation formalism. DEVS is an attractive methodology because of its generality and simplicity, and thus it has been extended in a number of directions, including: nonstandard logics [10], adaptive model structure [2], distributed and parallel implementations [24], Markov processes [1, 20], mixture with discrete time methods for continuous systems modeling [16, 23], and nondeterministic time-advance functions and symbolic time bases [31].

One very attractive feature of DEVS is its closeness to mathematical systems theory, and in particular finite process and automata theory [29]. Fishwick and Zeigler have argued [9] that such systems-theoretically based modeling methods can provide a more appropriate base to develop Qualitative Modeling (QM) [8] than the traditional qualitative physics bases.

This work generally is intended to move in that direction, while not dismissing the important issues brought up by traditional QM approaches [15]. This paper will describe extensions to DEVS based on some new ideas which relate general process and automata theory with General Information Theory (GIT) [18] and QM. Specifically, we will consider the applicability of possibilistic and fuzzy processes and automata to the DEVS methodology.

2 The Process Theoretical View of Classical DEVS

We first define DEVS and cast them in process theoretical terms.

2.1 DEVS

Following Zeigler [30], we have:

Definition 1 (Discrete EVent System (DEVS)) Let $\mathcal{D} := \langle X, Y, S, t, \delta_{\rm int}, \delta_{\rm ext}, \lambda \rangle$, where:

- X is a set of external events;
- Y is a set of outputs;
- S is a set of states;
- $t \colon S \mapsto [0, \infty]$ is the time advance function, where $[0, \infty] := \mathbb{R}^+ \cup \{\infty\}$;
- $\delta_{\rm int} \colon S \mapsto S$ is the internal transition function;

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- $\delta_{\text{ext}}: Q \times X \mapsto S$ is the external transition function, where

$$\begin{aligned} Q &:= \{\langle s, e \rangle : s \in S, 0 \leq e \leq t(s)\} \\ &= \bigcup_{s \in S} [0, t(s)] \subseteq S \times [0, \infty] \end{aligned}$$

  is the total state set; and

- $\lambda: S \mapsto Y$ is the output function.

These functions are interpreted as follows. Assume that $\mathcal{D}$ is in state $s \in S$. If an event $x \in X$ arrives after a duration $e \leq t(s)$, then $\mathcal{D}$ transits to state $\delta_{\text{ext}}(\langle s, e \rangle, x)$. If no event arrives before $t(s)$, then s is said to expire, and $\mathcal{D}$ transits to state $\delta_{\text{int}}(s)$. Finally, at all times, $\mathcal{D}$ produces output $\lambda(s)$.

2.2 Classical Processes

We now adapt the following classical definitions of finite processes and automata from Hopcroft and Ullman's standard text [11].

Definition 2 (Finite Processes and Automata) Assuming the following sets and functions:

State Set and Initial State: S and $s_0 \in S$;

Inputs and Outputs: X and Y;

State Transition Function: $\delta: S \times X \mapsto S$;

Output Function: $\lambda: S \times X \mapsto Y$;

then define the following systems:

Mealy Automaton: $A := \langle S, s_0, X, Y, \delta, \lambda \rangle$

Moore Automaton: Given A, if $\forall s \in S, \forall x_1, x_2 \in X, \lambda(s, x_1) = \lambda(s, x_2)$, then define $A^O := \langle S, s_0, X, Y, \delta, \lambda \downarrow X \rangle$, where $\lambda \downarrow X: S \mapsto Y$ is the projection of λ through X: $\forall s \in S, (\lambda \downarrow X)(s) := \lambda(s, x_0)$, for any arbitrary $x_0 \in X$.

Finite Automaton: Given A^O, if $Y = \emptyset$, then define $A^F := \langle S, s_0, X, \delta \rangle$;

Finite Process: Given A^F, if $X = \emptyset$, then define $Z := \langle S, s_0, \delta \downarrow X \rangle$.

A Mealy automaton is interpreted as a full automaton; the outputs of a Moore automaton depends entirely on the state, and not on the inputs; a finite automaton has no outputs; and a finite process has no inputs.

2.3 Process-Theoretical View of DEVS

Clearly DEVS share a great deal in common with classical automata. But classical automata, finite state machines, and sequential processes differ from DEVS in that DEVS have a continuous-time role played by the time advance function $t(s) \in [0, \infty]$. By focusing on the discrete aspect of DEVS, that is the discrete *events*, we can cast DEVS to some extent in finite process-theoretical terms.

Definition 3 (DEVS Moore Automaton) Given a DEVS $\mathcal{D}$, and an $s_0 \in S$, then the system $A^O_{\mathcal{D}}(s_0) := \langle S, s_0, X^*, Y, \delta^*, \lambda \rangle$ is the Moore automata constructed from $\mathcal{D}$ with initial state s_0, where: S, s_0, Y, and λ are as above;

- $X^* := (X \times [0, \infty)) \cup \{T\}$ is the extended input alphabet; and
- $\delta^*: S \times X^* \mapsto S$ is the extended state transition function, with $\forall s \in S, \forall x^* \in X^*$,

$$\delta^*(s, x^*) := \begin{cases} \delta_{\text{int}}(s), & x^* = T \\ \delta_{\text{ext}}(\langle s, e(x) \rangle, x), & x^* = \langle x, e(x) \rangle \end{cases} .$$

$A^O_{\mathcal{D}}(s_0)$ is interpreted as follows:

- The input alphabet X is extended in two ways:
  1. Each input event $x \in X$ which might occur is paramaterized by its time of occurrence $e(x) \in [0, \infty)$;
  2. The special event T is added;
- The internal and external transition functions δ_{int} and δ_{ext} are combined into the new extended transition function δ^*. $A^O_{\mathcal{D}}(s_0)$ necessarily receives an input $x^* \in X^*$, which is either:
  1. An event $x \in X$ at time $e(x) \leq t(s)$, triggering the external state transition to $\delta_{\text{ext}}(\langle s, e(x) \rangle, x)$; or
  2. The special event $T \in X^*$, indicating that s has expired, and triggering the internal state transition to $\delta_{\text{int}}(s)$.

Because of the continuous-time nature of DEVS they cannot be described purely in classical finite automata-theoretical terms. Thus $A^O_{\mathcal{D}}(s_0)$ is not a complete representation of $\mathcal{D}$. Instead, $A^O_{\mathcal{D}}(s_0)$ is the Moore automaton embedded within a larger system which will complete the representation of $\mathcal{D}$. This larger environment is responsible for interpreting the time advance function t so that:

- Any incoming event $x \in X$ occurs at an acceptable time $e(x) \leq t(s)$; and
- When s expires, then the special event $T \in S^*$ is sent to $A^Q_{\mathcal{D}}(s_0)$.

3 GIT for Qualitative Modeling

3.1 Qualitative Modeling Methods

Classical DEVS are entirely deterministic: a DEVS persists in a specific state, and, either on receiving an input event or expiring, transits to another specific state. There are situations when such specific and deterministic models are problematic, for example when:

- there is poor knowledge about the system being modeled or it is incompletely specified;
- there are missing or incomplete data;
- systems have parameters or states that are not known with certainty;
- or complexity makes detailed prediction difficult.

Under these circumstances QM techniques [3, 8], broadly described as the attempt to model systems at deliberately high levels of abstraction, can help. Of course, these approaches produce models that are less precise than they might be, but with the tradeoff of potentially greater tractability and accuracy (the less you say, the better your chance of being right).

When looking to provide a qualitative capability in DEVS, it is natural to turn to the generalizations of processes and automata which involve uncertainty. Traditional extensions include nondeterministic [11] and stochastic [26] processes. These map directly to one of the most prominent QM methods, which involve placing uncertainty distributions on state variables.

Here the uncertainty about some attribute is represented mathematically by weights on all possible values. The set of weights, as a distribution, acts as a meta-state in the space of all such distributions, and functional equations relating these meta-states produce predictions about the meta-state distribution at future times. The most familiar of these models are Markov processes, Bayesian networks, and other kinds of stochastic models [21].

3.2 Possibility Theory in GIT

But recent years have seen a proliferation of new, non-probabilistic mathematical methods for the representation of uncertainty and information in systems models. Following Klir [18], we call these methods collectively "General Information Theory" (GIT), which includes fuzzy sets, systems, and logic [19]; fuzzy measures [28]; random set [17] and Dempster-Shafer evidence theory [25]; possibility theory [6]; and others. Each of these involves some form of generalization or extension away from stochastic representations, and probability is retained in GIT as a limiting case.

Within GIT, possibility theory stands out, because although it is logically independent from probability, they exist in parallel and are formally analogous [4]: probability and possibility measures arise in Dempster-Shafer evidence theory as fuzzy measures defined on random sets; possibility and their dual necessity measures represent extreme ranges of probability intervals; and the distributions of all these generally non-additive fuzzy measures are fuzzy sets.

A detailed description of fuzzy and possibility theory is not possible here (see [5, 14, 19]), but I will introduce some basic concepts. Assume a finite universe $\Omega = \{\omega_i\}, 1 \leq i \leq n$. A function $\nu\colon 2^\Omega \mapsto [0,1]$ is a (finite) fuzzy measure [28] if $\nu(\emptyset) = 0$ and $\forall A, B \subseteq \Omega, A \subseteq B \rightarrow \nu(A) \leq \nu(B)$. A fuzzy subset denoted $\widetilde{A} \,\widetilde{\subseteq}\, \Omega$ is defined by its membership function $\mu_{\widetilde{A}}\colon \Omega \mapsto [0,1]$. $\mu_{\widetilde{A}}(\omega_i)$ indicates the extent to which $\omega_i \in \widetilde{A}$, with $\mu_{\widetilde{A}}(\omega_i) = 1$ indicating complete inclusion and $\mu_{\widetilde{A}}(\omega_i) = 0$ indicating complete exclusion.

A possibility measure Π is a fuzzy measure where $\forall A, B \subseteq \Omega, \quad \Pi(A \cup B) = \Pi(A) \vee \Pi(B)$ and $\vee$ is the maximum operator. Then the possibility distribution is a function $\pi\colon \Omega \mapsto [0,1]$, with $\pi(\omega_i) = \Pi(\{\omega_i\})$. In reverse, $\forall A \subseteq \Omega, \Pi(A) = \bigvee_{\omega_i \in A} \pi(\omega_i)$, and for normalization $\Pi(\Omega) = 1 \leftrightarrow \bigvee_{i=1}^n \pi(\omega_i) = 1$. So to some extent it is justified to view possibility theory as similar to probability theory where $+$ and $\times$ are replaced by $\vee$ and $\sqcap$ (although this risks oversimplification). $\sqcap$ remains as a free parameter.

Further components of possibilistic systems theory include possibilistic measures of information, which are called nonspecificities and are strict correlates to stochastic entropies, and possibilistic correlates of most of the constructs of stochastic systems theory, including marginals, joints, and conditionals, possibilistic Bayesian networks, and possibilistic processes and automata [14].

4 General Process Theory and GIT

Classical processes and automata can be generalized in the context of GIT to a class of general automata defined as temporal processes with input and output where states are valued on a lattice and the transition function is represented by an algebraic semiring. Depending on the choices made for the semiring operators, valuation lattice, and normalization conditions, the traditional classes of deterministic, nondeterministic, and stochastic (Markov) processes are recovered. Other choices result in the newer classes of fuzzy and possibilistic (possibilistic Markov) processes [14], both of which generalize nondeterministic processes better than stochastic processes do.

4.1 General Processes

Following Peeva [22] and Joslyn [14] we have:

Definition 4 (General Process) A system $\mathcal{Z} := \langle S, \phi^0, V, \mathcal{R}, \Delta \rangle$ is a general process if:

- V is the valuation set, a lattice with $0, 1 \in V$. Here we assume that V is a chain with $V \subseteq [0,1]$;
- $\mathcal{R} = \langle \sqcup, \sqcap \rangle$ is a conorm semiring, so that:
  - $\sqcup \colon V^2 \mapsto V$ (resp. $\sqcap \colon V^2 \mapsto V$) is a triangular conorm (resp. norm): an associative, commutative, monotonic operator with identity 0 (resp. 1);
  - $\sqcap$ distributes over $\sqcup$;
- $\Delta \colon S^2 \mapsto V$ is the transition function; and
- $\phi^\tau \colon S \mapsto V$ are a family of state functions for $\tau \in \{0, 1, \ldots\}$, with:
  - ϕ^0 a given initial state function; and
  - $\forall s \in S, \tau > 0$,

$$\phi^\tau(s) := \bigsqcup_{s' \in S} \phi^{\tau-1}(s') \sqcap \Delta(s, s'). \qquad (5)$$

When S is finite with $S = \{s_i\}, 1 \le i \le n := |S|$, then it is common to consider:

- ϕ^τ as the vector $\vec{\phi}^\tau = \langle \phi_i^\tau \rangle$, with $\phi_i^\tau := \phi^\tau(s_i)$;
- Δ as a matrix $\Delta = [\Delta_{ij}]$ for $1 \le i, j \le n$, with $\Delta_{ij} := \Delta(s_i, s_j)$; and
- $\vec{\phi}^\tau = \vec{\phi}^{\tau-1} \circ \Delta$ where $\circ$ is matrix composition over the semiring $\mathcal{R}$, as shown in (5).

General processes act as generalized first order stationary Markov processes. The ϕ^τ act as distributions on the states s_i, giving a weight in $[0,1]$ to each state at each time τ. Similarly, $\Delta(s_i, s_j)$ is the weight on the transition from state s_j to state s_i. (5) then determines future state vectors from past state vectors and Δ. Depending on the particular properties of V and $\mathcal{R}$, general processes might take on different forms, including classical finite processes as a special case. These will be considered below.

Definition 6 (Normal Processes) Assume a general process $\mathcal{Z}$. Then:

- ϕ^t is normal if $\bigsqcup_{s \in S} \phi^t(s) = 1$;
- Δ is transition normal if $\forall s' \in S, \bigsqcup_{s \in S} \Delta(s, s') = 1$;
- $\mathcal{Z}$ is normal if Δ is transition normal and $\forall t \ge 0, \phi^t$ is normal.

Theorem 7 (Process Normality) [14] For a general process $\mathcal{Z}$, if ϕ^0 is normal and Δ is transition normal, then $\mathcal{Z}$ is normal.

4.2 Special Processes

A number of cases follow depending on the specializations made for $\mathcal{R}, V$, and normalization. These are summarized in Tab. 1 and Fig. 1, where for $x, y \in [0,1], x +_b y := (x + y) \wedge 1$ is the bounded sum operator and $\wedge$ is the minimum operator. In the figure, non-degeneracy is simply the condition that $\forall \tau, \exists s_i, \phi_i^\tau > 0$; and certainty is the condition for deterministic systems described below. We now describe these classes more specifically:

Stochastic: When $\mathcal{R} = \langle +_b, \times \rangle$ is an additive semiring, then $\mathcal{Z}_p$ is a stochastic process. Now $p^\tau(s_i) := \phi^\tau(s_i) \in [0,1]$ is the probability of being in state s_i at time τ; Δ is a stochastic matrix with $p^\tau(s_i | s_j) := \Delta_{ij}$ the conditional probability of transiting from state s_j to state s_i; and $\circ$ is normal matrix composition $\cdot$. Here normalization by $+$ is required, so that $\forall \tau, \sum_i p_i^\tau = 1$. This implies the weaker conorm $+_b$ normalization $(\sum_i p_i^\tau) \wedge 1 = 1$.

Fuzzy: When $\mathcal{R} = \langle \vee, \sqcap \rangle$ for any norm $\sqcap$ then $\widetilde{\mathcal{Z}}$ is a fuzzy process [27]. Now $\vec{\mu}^\tau := \vec{\phi}^\tau \mathrel{\widetilde{\subseteq}} S$ is the fuzzy set representation of the overall meta-state; $\mu^\tau(s_i) := \phi^\tau(s_i) \in [0,1]$ is the degree of membership of s_i in the set of states of $\widetilde{\mathcal{Z}}$; $\Delta \mathrel{\widetilde{\subseteq}} S^2$ is now a fuzzy matrix representing a fuzzy relation of the fuzzy linkage between the prior state s' and the subsequent state s; and $\circ$ is fuzzy matrix composition [19]. Note that there is no normalization, and all values are unconstrained over $[0,1]$.

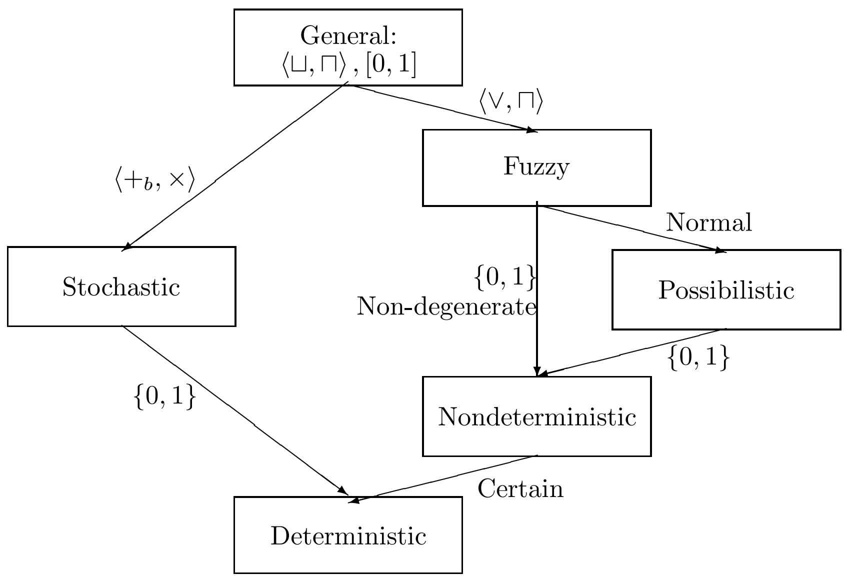

Figure 1: Relations among classes of processes

Nondeterministic: Given a fuzzy process $\widetilde{\mathcal{Z}}$, if V is restricted to $\{0,1\} \subseteq [0,1]$, then $\mathcal{Z}_n := \widetilde{\mathcal{Z}}$ is a classical nondeterminstic process [11], so that at time τ $\mathcal{Z}_n$ exists in a set of possible states and any state can transit to multiple states.

Deterministic: Given either a stochastic process $\mathcal{Z}_p$ with $V = \{0,1\}$, or a nondeterministic process $\mathcal{Z}_n$ with the certainty requirement $\forall \tau, \exists! s_i, \mu^t(s_i) = 1$, then $\mathcal{Z}_d$ results as a classical deterministic process [11]. $\mathcal{Z}_d$ is always in one definite state, and transits to another definite state. We have:

Theorem 8 [14] Each deterministic process $\mathcal{Z}_d$ maps to a unique finite process Z from (2), and vice versa.

Possibilistic: Finally, given a fuzzy process $\widetilde{\mathcal{Z}}$ which is normal by $\vee$, then $\mathcal{Z}_\pi := \widetilde{\mathcal{Z}}$ is a possibilistic process [13, 14]. Now $\pi^\tau(s_i) := \phi^\tau(s_i) \in [0,1]$ is the possibility of being in state s_i at time τ; Δ is called a possibilistic matrix $\mathbf{\Pi} := \Delta$, with $\pi^\tau(s_i|s_j) := \mathbf{\Pi}_{ij} = \Delta_{ij}$ being the conditional possibility of transiting from state s_j to state s_i; and $\circ$ is again fuzzy matrix composition.

4.3 Possibilistic Processes

The important new class of possibilistic processes was introduced by Joslyn [13, 14] (see also Janssen *et al.* [12]). They are essentially possibilistic correlates of first-order, stationary Markov processes, and mark the appearance of a new form of qualitative process.

An example possibilistic process is shown in Fig. 2 for $S = \{x, y, z\}$. The arcs indicate state transitions, each of which is non-additively weighted with the conditional possibility of the transition. So, for example, $\pi(x|y) = .8$, and the possibilistic transition matrix is

$$\mathbf{\Pi} = \begin{bmatrix} 0.0 & 0.8 & 0.0 \\ 1.0 & 0.0 & 0.0 \\ 0.2 & 1.0 & 1.0 \end{bmatrix}.$$

$\mathbf{\Pi}$ is transition normal, with a 1 in each column, or equivalently each node has an out-arc labeled 1. Nodes may have multiple out-arcs, however, so that a unitary weight does not indicate a *necessary*, but rather a *completely possible* transition. For example, from state x, transition to z is .2 possible, but transition to y is also (completely) possible. z is an absorbing state, with self-transition the only possibility.

Let $\sqcap = \wedge$, and assume that at time $\tau = 0$ the system begins *definitely* in state x, so that $\vec{\pi}^0 = \langle 1, 0, 0 \rangle^T$, which is normal. Then future state vectors are

τ	0	1	2	3	4
$\vec{\pi}^\tau$	1.0	0.0	0.8	0.0	0.8
	0.0	1.0	0.0	0.8	0.0
	0.0	0.2	1.0	1.0	1.0

(7) guarantees that all state vectors are normal. The process settles into a normal element of state z, which acts as a kind of attractor or absorbing state. But

Class	**Denotation**	$\mathcal{R}$	V	**Normal**
Stochastic	$\mathcal{Z}_p$	$\langle +_b, \times \rangle$	$[0, 1]$	Yes
Fuzzy	$\widetilde{\mathcal{Z}}$	$\langle \vee, \sqcap \rangle$	$[0, 1]$	Not necessarily
Nondeterministic	$\mathcal{Z}_n$	$\langle \vee, \sqcap \rangle$	$\{0, 1\}$	Yes
Deterministic	$\mathcal{Z}_d$	$\langle +_b, \times \rangle = \langle \vee, \sqcap \rangle$	$\{0, 1\}$	Yes
Possibilistic	$\mathcal{Z}_\pi$	$\langle \vee, \sqcap \rangle$	$[0, 1]$	Yes

Table 1: Special cases of processes.

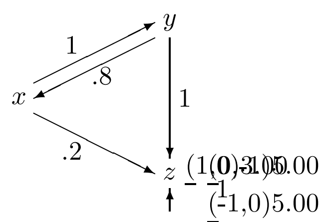

Figure 2: Weighted state transition diagram for a possibilistic process.

in virtue of the cycle between x and y with transition possibilities of 0 and .8 respectively, each of those states in turn remains .8 possible. State y can never recover the 1 possibility it has at $t = 1$, because the system is never again unequivocally in state x.

4.4 General Automata

From (2), finite processes are at the heart of the automata models, including DEVS. Similarly, general processes can be extended to general automata.

Definition 9 (General Automata) Given a general process $\mathcal{Z} = \left\langle S, \phi^0, V, \mathcal{R}, \Delta \right\rangle$ and input and output alphabets X and Y, extend the state transition function to $\Delta' \colon S \times X \times S \mapsto V$, and introduce a general output function $\Lambda \colon S \times X \times Y \mapsto V$. Then define:

General Mealy Automaton:
$\mathcal{A} := \left\langle S, \phi^0, X, Y, \Delta', \Lambda, V, \mathcal{R} \right\rangle$ where given that $x \in X$ is the current input then ϕ^τ is extended to

$$\phi'^{\tau}(s) := \bigsqcup_{s' \in S} \phi'^{\tau-1}(s') \sqcap \Delta'(s, x, s').$$

General Moore Automaton: Given $\mathcal{A}$, if

$$\mathop{\forall}_{s \in S} \mathop{\forall}_{x_1, x_2 \in X} \mathop{\forall}_{y \in Y} \Lambda(s, x_1, y) = \Lambda(s, x_2, y)$$

then define $\mathcal{A}^O := \left\langle S, \phi^0, X, Y, \Delta', \Lambda \downarrow X, V, \mathcal{R} \right\rangle$;

Finite Automaton: Given $\mathcal{A}^O$, then if $Y = \emptyset$, then define $\mathcal{A}^F := \langle S, \phi^0, X, \Delta', V, \mathcal{R} \rangle$.

In these systems inputs and outputs are added. The inputs essentially paramaterize the weighted state transition function, or can be seen as providing a set of transition matrices $\Delta'(x)$, one for each input symbol.

Similarly, the output function $\Lambda(s, \cdot, y)$ in the general Mealy automaton is a set of matrices $\Lambda(x)$ paramaterized by $x \in X$ recording the conditional weight of outputting y given being in state s. The final output distribution at time τ is then given by $\Lambda(x) \circ \phi'^{\tau}$. As discussed in Sec. 4.2, depending on the specialization of $\mathcal{R}$ and V, these weights may be fuzzy, stochastic, nondeterministic, deterministic, or possibilistic.

5 Process General DEVS

It is clear that these ideas are directly applicable to the definition of a general process theoretical extension of a classical DEVS. We will do this by extending the Moore automaton of a DEVS $A^O_{\mathcal{D}}(s_0)$ to a general Moore automaton.

Definition 10 (DEVS General Moore Automaton) Assume a general Moore automaton $\mathcal{A}^O_{\mathcal{D}} = \left\langle S, \phi^0, X^*, Y, \Delta'^*, \Lambda, V, \mathcal{R} \right\rangle$ where:

- X^* is an extended input alphabet as in (3);
- $\Delta'^* \colon S \times X^* \times S \mapsto V$ is the set of transition matrices parameterized by the $x \in X$ as in (9); and
- We denote here $\Lambda \colon S \times Y \mapsto V$ as the simple weighted output function which is not dependent on the input x, as in the Moore automaton of (9).

Following (3) in reverse, we can now construct a generalized DEVS by distinguishing state transitions triggered by event inputs $\langle x, e(x)\rangle \in X^*$ and those due to internal transitions for $T \in X^*$. Thus we arrive at the following definition of a general DEVS.

Definition 11 (General DEVS) A general DEVS is a system $\hat{\mathcal{D}} := \langle X, Y, S, t, \Delta_{\rm int}, \Delta_{\rm ext}, \Lambda, V, \mathcal{R}\rangle$, where: X, Y, S, t, Λ, V, and $\mathcal{R}$ are as above (S finite);

- $\Delta_{\rm int}\colon S^2 \mapsto V$ is the general internal transition function;
- $\Delta_{\rm ext}\colon Q \times X \times S \mapsto V$ is the general external transition function, with Q as above in (1).

These are interpreted as follows. Assume that $\hat{\mathcal{D}}$ is unequivocally in a certain state $s_0 \in S$, represented as the vector $\vec{\phi}$ with $\phi_0 = 1$ and $\forall s_i \neq s_0, \phi_i = 0$. If an event $x \in X$ arrives after a duration $e \leq t(s)$, then $\hat{\mathcal{D}}$ transits to the new state vector $\vec{\phi}' = \Delta_{\rm ext}(x) \circ \vec{\phi}$. If no event arrives before $t(s)$, then $\hat{\mathcal{D}}$ transits to the new state vector $\vec{\phi}' = \Delta_{\rm int} \circ \vec{\phi}$. Finally, $\mathcal{D}$ outputs a weighted vector of output symbols given by $\Lambda \circ \vec{\phi}$.

6 Future Work

These ideas can be continued different directions:

- This formulation separates the Moore automaton core of a DEVS from the continuous aspect of the time of the input events. Thus the input events are not given general weights, that is events $x \in X$ occur unequivocally. Extension to weighted input events remains to be done.
- Similarly, we do not deal with coupled general DEVS. This development depends on the above weighting of inputs, since in a coupled DEVS the outputs of one DEVS, which *are* weighted, are fed to the inputs of another, which have not been.
- Ahn and Kim [1] and Melamed [20] have also considered autonomous DEVS as stochastic processes. It would be useful to compare our approach with theirs, and to consider generalization of their approach to fuzzy, possibilistic, or other forms.
- Classical DEVS input and output segments are intervals, which generalize naturally to fuzzy intervals [7], which are both generalizations of classical intervals and kinds of possibility distributions. Here the membership grade would indicate the extent to which a certain time point is included in the interval, and thereby the extent to which a certain event or transition occurs.
- Finally, Joslyn and Henderson [16] have suggested that weights can be placed on the linkages among the components of a coupled DEVS, indicating some information about the degree to which the linkage holds. The variety of automata models is then available depending on the choices of valuation lattice, semiring operators, and normalization.

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