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high-dimensional spaces is computationally extremely challenging. In this talk we present ExSpliNet, an interpretable and expressive neural network model that combines ideas of Kolmogorov neural networks, ensembles of probabilistic trees, and multivariate B-spline representations. The new model is a feasible generalization of the B-spline neural network (BSNN) model towards high-dimensional data relying on a Kolmogorov-like structure to avoid the use of high-variate tensor-product B-splines while still maintaining expressive power. Furthermore, we carry out a theoretical study of the universal approximation properties of ExSpliNet. The main ingredients of the proof are the Kolmogorov superposition theorem and classical approximation estimates for multivariate splines. Finally, we illustrate the suitability of the proposed model to address data-driven function approximation and to face differential problems, in the spirit of physics-informed neural networks (PINNs). We also show the general applicability of the model for classical machine learning tasks like image classification and regression.

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## CP17

### Topological Analysis of Temporal Hypergraph Data

Hypergraphs extend graphs to include high-order edges recording multi-way relationships. Graphs are hypergraphs with uniform edge size two, but hypergraphs can have edges of arbitrary size. They are closely related to topological structures like abstract simplicial complexes, which are themselves hypergraphs including all sub-edges, and can have non-trivial topological properties. Simplicial homology identifies the topological structure of hypergraph data by measuring the dimensions of their homology groups (Betti sequences), and thus their open cycles of different dimension  $k$ , including connected components as a special case for  $k = 0$ . Hypergraph structures commonly carry additional data, including time intervals on edges, so the topological properties of such temporal hypergraphs can change over time. A sliding window over the temporal intervals of a THG identify a temporal sequence of sub-hypergraphs, and thus a changing sequence of Betti sequences. While useful, a more sophisticated approach called zigzag persistent homology links the sequential sub-hypergraphs to not only identify common structures between sample windows, but how much they persist over all the data. We demonstrate our method first on synthetic data, and then on a collection of Reddit threads, tracking users against threads in the context of their time stamps. A complex temporal pattern is revealed for dimensions  $k = 0, 1, 2$ . Comparison to non-topological hypergraph properties is provided.

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## CP18

### Nonlocal Models for Deep Neural Networks

Recently, continuous-depth models of neural networks have shown a series of advantages in machine learning, which include memory costs that do not increase with the number of layers (a major bottleneck in training deep models), parameters efficiency, as well as obtaining continuous time series models that can naturally incorporate data at arbitrary times. In this talk I will present some continuous-depth models that also incorporate nonlocality with respect to the layer and discuss their performance and advantages. The results are backed by theoretical studies into well-posedness of solutions for the integro-differential models, as well as stability results which are important in designing robust DNNs that can withstand adversarial attacks

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## CP18

### Examining Stiffness in ResNets through Interpretation as Discretized Neural ODEs

Neural Ordinary Differential Equations (NODEs) emerge as the limit of Residual Neural Networks (ResNets) as the number of layers tends to infinity. While NODEs are successfully applied in practice to a range of machine learning problems, studying their behavior can provide insights into the role of depth in network architecture. We expect that mathematical insights and legacy knowledge of ODE analysis can be applied to NODEs to better understand the behavior of neural networks, improve their generalization, and develop more efficient training algorithms. Drawing from ODE theory, we focus on the concept of stiffness and consider the analogue of stiffness for ResNets. We first develop a heuristic for quantifying the stiffness of a ResNet given an input training set. We then study the use of this stiffness measure as a penalizing regularizer during training, and examine its effects on generalization.

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## CP18

### Differentiable Computational Fluid Dynamics As a